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The Demand for Money in the United States, 1959.1-2013.11

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Abstract

The purpose of this paper is to estimate a stable long run money demand function for the United States using monthly data from January 1959 till November 2013. The paper has three features. One, the sample includes the recent period of near-zero values of the short term interest rate. Two, the monetary aggregate that is selected is the MZM money stock, as compiled by the Saint Louis Fed. Three, the data is monthly. One crucial question that is asked is whether the interest rate variable should enter in a log functional form or not. This depends on the stability of the constrained and the unconstrained money demand functions over the whole period. When the coefficients on the scale and price level variables are constrained to be unitary the best specification is obtained by including the interest rate variable as is. When all constraints are relaxed the best specification is when the interest rate variable enters in logs. There is evidence that the imposed constraints do not hold well statistically. The main conclusion is therefore that a full log-log model of money demand is the most appropriate, although there is some evidence of non-linearity in the relation. This implies that, at very low interest rates, there is a Keynesian liquidity trap and that, consequently, monetary policy may become totally ineffective.
Keywords: Money demand, United States of America, constrained and unconstrained functional forms, modeling the interest rate variable, long run monthly data, MZM money supply, unit roots, cointegration, liquidity trap

1. Introduction

The subject of money demand is a branch in economics that is of rather recent history. It is true that the classics believed that the money stock and the price level vary positively together in the long run but they did not formulate any short run association. Keynes named the money demand function liquidity preference but he was convinced that this function to be inherently unstable. The thrust for a stable money demand function arose from the efforts of Milton Friedman, starting in the 1950s, at the University of Chicago (Friedman, 1956; Friedman and Schwartz, 1962, 1982). A proper model for money demand is quite valuable. First, money demand is one of the tenets of the IS/LM Keynesian framework despite the doubts of Keynes on its stability. Without it this framework can only be partially applied or studied if ever. Second, a stable money demand function is necessary for the conduct of monetary policy by a central bank, and for the choice between a money supply rule, that targets the money supply, and an interest rate rule, that targets the interest rate (Poole, 1970). Third, money demand is a structural equation that is posited to hold in many macroeconomic models besides IS/LM. Fourth, it is universally recognized that money demand has both a short run and a long run behavior. The two effects need to be separated. Fifth, the way the underlying variables are anchored together is economically vital. Finally, the usual demand for money function asserts implicitly that real balances are endogenous, while monetary policy is typically conducted through controlling exogenously the money supply. Understanding and documenting the variables that are weakly exogenous from others that are endogenous is therefore vital, and this is possible by resorting to appropriate econometric procedures. Other appropriate econometric procedures can help in determining how money demand moves in the long run.

The paper is organized as follows. In the following section, section 2, a brief survey of the literature since the early of the 21st century is provided. Section 3 presents the theoretical and modeling background. The empirical results are in section 4. The paper follows the advice of Ireland (2009) by using a different US money stock than the one used commonly in the literature, and by including in the analysis recent data on near-zero short term interest rates. It is true, however, that the MZM money stock, used in this paper, has also been used elsewhere (Motley, 1988; Poole, 1991; Duca and VanHoose, 2004; Teles and Zhou, 2005). Section 5 summarizes and concludes.

2. The survey of the literature

Ball (2001) extends the period covered by Stock and Watson (1993), and, contrary to Stock and Watson, uncovers a stable money demand function for the US that features an income elasticity of around 0.5 and an interest rate semi-elasticity of around -0.05.

Hondroyiannis et al. (2001) study the stability and interest-sensitivity of long run money demand in the US for annual data going back to 1870. Contrary to other authors, and commensurate with the results in this paper, they estimate the interest elasticity to be relatively low, between -0.04 and -0.15.

Oh (2002) finds that during the period between January 1961 and December 1992 the demand for money is highly stable when the money stock M1 is used.

Sarno et al. (2003) estimate a nonlinear equilibrium correction model for real balances, using annual data for the US going back to 1869. See also Minford (2004) where a non-linear money demand relation is uncovered for the United Kingdom. Sarno et al. (2003) argue that since the examined period has witnessed many fundamental changes in the key variables affecting money demand it is unlikely that the latter function will turn out to be stable and linear over the whole period. Their analysis is correct and they find empirical evidence for nonlinearity where the non-linearity depends on the speed of adjustment to the long run. In this paper a different non-linear relation that is based on calendar breaks receives some support although the hypothesis of a stable and linear money demand function over a relatively long period from 1959 till 2013 receives also ~~some~~ support.

Schmidt (2004) reexamines the significance of the interest rate variable in money demand. He finds the interest rate to be strongly exogenous, not influenced by either lagged variables or by the cointegration lagged residual. He also rejects a unitary coefficient for the price level variable.

Teles and Zhou (2005) try to find out which measure of money is the most appropriate. They conclude that the money stock M1 was good enough until the end of the 1970s, after which time the MZM money stock becomes more relevant. They estimate the interest elasticity to be between -0.20 and -0.26.

Dutkowsky et al. (2006) define new narrow money stocks for the US, named by them M1RS and M1S, which take into consideration the reclassification by banks of demand deposits into instruments with zero statutory reserve requirements. This enables banks to avoid reserve requirements. They find in general that the newly defined money stocks provide for more stable money demand relations. They also use the MZM money stock in their analysis.

Haug and Tam (2007) analyze the money demand relation relative to the monetary base (M0) and to the money supply stocks M1 and M2. They are interested to know which specification is better: including the interest rate variable in level or in log. They also consider a non-linear error-correction model with smooth transition. Cointegration, or a long run relation, is found when the opportunity cost of money is the short term interest rate. The interest rate elasticity is measured to be -0.49 for M0, -0.40 for M1, and -0.29 for M2. For the post war data the best model is that for the monetary base with the inclusion of the log of the interest rate variable.

Schmidt (2008) estimates a comprehensive four-equation model of money demand and money supply. Particularly significant is Schmidt's rejection of a money demand price elasticity of one and his support for a price elasticity that is less than one. This is consistent with the evidence in this paper. Also, consistent with this paper, Schmidt (2008) finds that the interest rate variable is weakly exogenous. However he finds that real income is also weakly exogenous which does not correspond to the evidence in this paper.

Choi and Jung (2009) study the stability of the US money demand function. They find that there is no stable long run relation between 1959 and 2000. Indeed two breaks exist, the first around April 1974 and the second around February 1986. And in each sub-sample the money demand function is stable. This paper finds other breaks.

Rao and Kumar (2011) estimate a money demand function for the US money stock M1on an annual basis. They include a trend in their equations and discover a break in 1998. But their estimate of the income elasticity is unitary which contradicts the finding in this paper.

3. The theory and the models

The theory of money demand spans already one century. The major issues tackled are the choice of variables in the demand for money equation, the functional forms, and the specification issues (Sriram, 1999). The earliest formulation is due to Fisher (1911), who started from the equation of exchange MV≡PY, which is an identity and where M is the nominal money stock, V is the velocity of circulation of money, P is the price level, and Y is income or wealth. Fisher's theory assumes that the velocity is constant and that output is fixed, leaving a proportionate relation between money and prices. Therefore, in the long run, money is neutral and does not affect real variables. Pigou (1917) and the Cambridge economists used the same relation but assumed only velocity to be constant. The result is that nominal money and nominal income move together. Later all other theories found that money is determined positively by a scale variable and negatively by an opportunity cost of money. Keynes (1953) separated money into transaction and precautionary on the one hand and speculative on the other hand. The first two are a function of a scale variable, while the third is a function of an interest rate variable. Keynes is an exception in the literature because, in his mind, the interest rate serves for speculation on bond prices and depends on a normal level of interest rates, and is not strictly speaking an opportunity cost of money. From then on demand for money was essentially considered to be demand for real money balances.

Baumol (1952) and Tobin (1956) use an inventory theoretic model for cash balances and demonstrate that the transaction portion of money depends also on the interest rate, contrary to what Keynes assumed. They find from their model that the income elasticity of money should be 0.5 and that the interest elasticity should be -0.5.

Tobin (1958) initiated a different theoretical approach by considering money as a store of value and by stressing the importance of the expected returns and risks of different financial assets, from a portfolio point of view, in the demand for money.

The monetarist tradition, exemplified by the works of Friedman that were referred to above, sees in the demand for money a demand like that for any other consumer commodity. The determinants of the demand for a consumer commodity and those of money should be the same. This also implies that real money is a function of a scale variable, and an interest rate variable, which is the price of money. But because of empirical irregularities the model is set into a partial adjustment framework where the lagged value of real money enters as an independent variable. This model became very popular in the 1970s but suffered from a specification problem and highly restrictive dynamics. In the 1980s buffer-stock models were more popular. These models include in the demand for money function an additional disequilibrium variable which is unanticipated money (Carr and Darby, 1981; Laidler, 1984). Unanticipated money is said to increase money holdings. More recently modeling relied on the micro foundations of monetary economics. In these models the scale variable is not aggregate income but aggregate consumption (Sargent, 1987; McCallum, 1989). In the 1990s the research was directed mainly to estimating long run cointegration relations and short run error-correction models. In the 2000s the stress was put mostly on the stability of the money demand function and especially on the existence of non-linearities.

The theoretical functional form starts from the most parsimonious model to the most unrestricted version. If M is the money stock, P is the price level, Y is a scale variable in real terms, r is the interest rate, and ln stands for the natural logarithm, then two specifications are of interest (Lucas, 2000; Ireland, 2009):

$$\ln M - \ln P - \ln Y = \alpha + \beta \ln r + \varepsilon \qquad \beta < 0 \tag{1}$$

$$\ln M - \ln P - \ln Y = \alpha^* + \beta^* r + \varepsilon^* \qquad \beta^* < 0 \tag{2}$$

where ε and ε^* are regression residuals, and α, β, α^*, and β^* are coefficients to be estimated. In this regard β is the interest rate elasticity of money demand, and β^* is the interest rate semi-elasticity of money demand. These two specifications assume that money demand is a demand for real money and that the income elasticity is unitary, which is exactly what is usually found in the empirical literature (Sriram, 2001). Since $\ln M, \ln P, \ln Y, \ln r$, and r are usually found to be integrated of order one, equations (1) and (2) assume implicitly that the first three variables are not cointegrated together, an assumption which is too strong to be acceptable because it means that there is no long run relation between these three variables. If there is a long run relation then the combination of the three variables will be integrated of order zero. Hence, this is contrary to standard knowledge. Especially noteworthy is that, since the left hand sides of equations (1) and (2) are non-stationary, this means that real balances and real income are not cointegrated, and therefore have no long run relation, something which is against the standard paradigm.

Equations (1) and (2) can be less restricted as follows:

$$\ln M - \ln P = \beta_0 + \beta_1 \ln Y + \beta_2 \ln r + \varepsilon \qquad \beta_1 > 0 \text{ and } \beta_2 < 0 \tag{3}$$

$$\ln M - \ln P = \gamma_0 + \gamma_1 \ln Y + \gamma_2 r + \varepsilon^* \qquad \gamma_1 > 0 \text{ and } \gamma_2 < 0 \tag{4}$$

where ε and ε^* are regression residuals, and $\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1$, and γ_2 are coefficients to be estimated. In this regard β_1 and γ_1 are the scale elasticities, β_2 is the interest rate elasticity of money demand, and γ_2 is the interest rate semi-elasticity of money demand. Equations (3) and (4) are the most researched functional forms of money demand. See, for example, the early papers by Baba et al. (1992) and McNown and Wallace (1992). Since $\ln M, \ln P, \ln Y, \ln r$, and r are usually found to be integrated of order one, equations (3) and (4) assume implicitly that $\ln M$ and $\ln P$ are not cointegrated, an assumption which is also too strong to be acceptable because it means that there is no long run relation between these two variables, and no room for long run neutrality of money. This is again contrary to theoretical standards, although equations (3) and (4) are the most common functional forms of money demand that are estimated in the empirical literature. Indeed there is a forceful argument that the dependent variable should be the log of real money balances. However instead of imposing this constraint ad hoc or otherwise a totally unconstrained version of money demand can be justifiable as follows (as in Miller, 1991):

$$\ln M = \beta_0 + \beta_1 \ln Y + \beta_2 \ln r + \beta_3 \ln P + \varepsilon \qquad \beta_1 > 0,\ \beta_2 < 0 \text{ and } \beta_3 > 0 \tag{5}$$

$$\ln M = \gamma_0 + \gamma_1 \ln Y + \gamma_2 r + \gamma_3 \ln P + \varepsilon^* \qquad \gamma_1 > 0,\ \gamma_2 < 0 \text{ and } \gamma_3 > 0 \tag{6}$$

where ε and ε^* are regression residuals, and $\beta_0, \beta_1, \beta_2, \beta_3, \gamma_0, \gamma_1, \gamma_2$, and γ_3 are coefficients to be estimated. There is a possibility that β_3 and γ_3 may not be unitary. This arises if prices are sticky even in the long run, or if prices have real long run effects on the economy. Equations (1), (3), and (5) make allowance for a liquidity trap while equations (2), (4), and (6) incorporate a satiety level for the interest rate. In addition the two specifications, especially the ones in equations (1) and (2), imply different welfare costs for the inflation rate (Lucas, 2000; Ireland, 2009).

4. The empirical results

All data are taken from the web site of the Federal Reserve Bank of Saint Louis. The selection for the money stock is the zero-maturity MZM money supply. The price level is the Consumer Price Index: All items for the US. The interest rate is the secondary market yield of the 3-month Treasury bill. The scale variable is the real disposable personal income. The data is monthly from the end of December 1958 to the end of March 2014 at the most, and from the end of January 1959 to the end of November 2013 at the least. All computations are carried out using the EViews 8 (2013) statistical package.

All the series are logged, except for the T-bill yield which is both logged and kept as is. In total there are 5 series to be tested for unit roots (Table 1). In addition there are two other series: the log of real money and the log of the ratio of real money to real disposable personal income. Two unit root tests are implemented: the Phillips and Perron test (1988) and the KPSS test (Kwiatkowski et al., 1992). The results are concordant except for one series, the continuously compounded inflation rate, or the change in the logs of the price level, which is found to be non-stationary according to the KPSS test but stationary according to the Phillips and Perron test. Otherwise all logged series are integrated of order one, and the changes in the logs are stationary, i.e. integrated of order zero. The T-bill yield and its change follow the same distributional properties as the logged series and their first-differences. Hence it can be rightly concluded that in general the seven variables are integrated of order 1, and consequently are stationary in first-differences. See Table 1.

Table 1. Unit root tests with a constant and a trend.

Variable X	Tests on X		Tests on Δ(X)	
	KPSS	PP	KPSS	PP
3-month US Treasury bill r	<1%	>10%	>10%	<1%
Log of the 3-month US Treasury bill $\ln r$	<1%	>10%	>10%	<1%
Log of the MZM money stock $\ln M$	<1%	>10%	>10%	<1%
Log of the Consumer Price Index: Total items for the U.S. $\ln P$	<1%	>10%	<1%	<1%
Log of the real disposable personal income $\ln Y$	<1%	>10%	>10%	<1%
Log of the ratio of money to the price level $\ln M - \ln P$	<1%	>10%	>10%	<1%
Log of the ratio of real money to real disposable personal income $\ln M - \ln P - \ln Y$	<1%	>10%	>10%	<1%

Notes: ln is the natural logarithm. The null hypothesis of the KPSS test is stationarity (Kwiatkowski et al., 1992). The null hypothesis of the PP test is non-stationarity (Phillips and Perron, 1988). The actual one-sided p-values of the PP test are retrieved from MacKinnon (1996). The sample size is monthly, from the end of December 1958 to the end of March 2014 at the most, and from the end of January 1959 to the end of November 2013 at the least.

Next cointegration tests are carried out following the methodology in Johansen (1988, 1991, and 1995) and in Johansen and Juselius (1990). These tests begin by the most restrictive specifications, which are equations (1) and (2) to the least restrictive, which are equations (5) and (6). The results for equation (1) are in Table 2.

Although the coefficient on the log of the interest rate is of a reasonable magnitude and has the correct sign the test fails to find any cointegration, or long run relation, between the log of the interest rate and the log of the ratio of real money on real disposable personal income by both the trace statistic and the maximum Eigen value statistic. Table 3 replaces the log of the interest rate variable with the same variable but without logs. The semi-elasticity of money demand with respect to the interest rate is negative, is statistically significant, and has a value of -120.8158. This high figure in absolute value is explainable by noting that the interest rate variable is divided by 1200 to get monthly decimal figures. Without this adjustment the semi-elasticity is -0.1007. The existence of one cointegration equation fails to be rejected by both the trace statistic and the maximum Eigen value statistic. This gives support to the specification in equation (2), i.e. including the interest rate variable as is, i.e. without logging it.

Table 2. Cointegration tests. The tested relation is: $\ln M - \ln P - \ln Y = \alpha + \beta \ln r$. T-statistic in parenthesis

α = -0.136059 β = -0.166031 (-2.28209)

Hypothesized number of cointegration equations	Eigen value	Trace statistic	5% critical value	Probability
None	0.007819	5.434018	15.49471	0.7611
At most 1	0.000433	0.284322	3.841466	0.5939

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 3.

Hypothesized number of cointegration equations	Eigen value	Maximum Eigen value statistic	5% critical value	Probability
None	0.007819	5.149696	14.26460	0.7228
At most 1	0.000433	0.284322	3.841466	0.5939

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 3.

Table 3. Cointegration tests. The tested relation is: $\ln M - \ln P - \ln Y = \alpha + \beta r$. T-statistic in parenthesis

α = -0.165683 β = -120.8158 (-6.17369)

Hypothesized number of cointegration equations	Eigen value	Trace statistic	5% critical value	Probability
None	0.027998	20.35142	15.49471	0.0085
At most 1	0.002575	1.694181	3.841466	0.1930

The actual p-values are retrieved from MacKinnon et al. (1999).The lag length is 2.

Hypothesized number of cointegration equations	Eigen value	Maximum Eigen value statistic	5% critical value	Probability
None	0.027998	18.65724	14.26460	0.0095
At most 1	0.002575	1.694181	3.841466	0.1930

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 2.

Table 4 estimates equation (3). The evidence is similar to equation (1). Although both the log of the scale variable and the log of the interest rate variable have coefficients with the correct sign, with the accurate magnitude, and with the needed statistical significance, the cointegration test fails to reject no-cointegration.

Table 4. Cointegration tests. The tested relation is: $\ln M - \ln P = \alpha + \beta_1 \ln Y + \beta_2 \ln r$. T-statistics in parenthesis

α = 0.292810 β_1 = 0.951961 (8.50442) β_2 = -0.178872 (-4.13606)

Hypothesized number of cointegration equations	Eigen value	Trace statistic	5% critical value	Probability
None	0.024227	25.80365	29.79707	0.1347
At most 1	0.014364	9.714918	15.49471	0.3033
At most 2	0.000341	0.223877	3.841466	0.6361

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 3.

Hypothesized number of cointegration equations	Eigen value	Maximum Eigen value statistic	5% critical value	Probability
None	0.024227	16.08873	21.13162	0.2197
At most 1	0.014364	9.491041	14.26460	0.2476
At most 2	0.000341	0.223877	3.841466	0.6361

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 3.

Equation (4), on the other hand, produces coefficients that are respectively 0.697303 for the scale variable, with a t-statistic of 12.5783, and -106.7706, or -0.0890 if divided by 1200, for the interest rate variable, with a t-statistic of -9.36718. Moreover the null of no-cointegration is rejected by the trace statistic with a p-value of 0.0275, but not by the maximum Eigen value statistic which has a p-value of 0.0820. Therefore the evidence is mixed for this specification. See Table 5.

Table 5. Cointegration tests. The tested relation is: $\ln M - \ln P = \alpha + \beta_1 \ln Y + \beta_2 r$. T-statistics in parenthesis

α = 2.715047 β_1 = 0.697303 (12.5783) β_2 = -106.7706 (-9.36718)

Hypothesized number of cointegration equations	Eigen value	Trace statistic	5% critical value	Probability
None	0.029319	31.98649	29.79707	0.0275
At most 1	0.018482	12.43559	15.49471	0.1372
At most 2	0.000273	0.179328	3.841466	0.6719

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 2.

Hypothesized number of cointegration equations	Eigen value	Maximum Eigen value statistic	5% critical value	Probability
None	0.029319	19.55091	21.13162	0.0820
At most 1	0.018482	12.25626	14.26460	0.1014
At most 2	0.000273	0.179328	3.841466	0.6719

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 2.

The most crucial equations are equations (5) and (6) which do not constrain the coefficients of the log of the scale variable and the log of the price variable to be both +1. Table 6 presents the results when the interest rate variable enters in logs. All three coefficients on the three independent variables have the correct sign, are of reasonable magnitude, and have the needed statistical significance. The null of no-cointegration is rejected by both the trace statistic and the maximum Eigen value statistic. The evidence supports the existence of only one cointegration relation. The income elasticity of money demand is estimated to be 1.757152, the price elasticity is estimated to be 0.245634, and the interest rate elasticity is estimated to be -0.038668. However the null hypothesis that the slope on the log of the scale variable is +1 is rejected by a likelihood ratio test at very low marginal significance levels, less than 0.00001. Moreover the null hypothesis that the slope on the log of the price level is +1 is also rejected by a likelihood ratio test at very low marginal significance levels, less than 0.00001. The joint null hypothesis that these two constraints hold exactly is naturally rejected at very low marginal significance levels, again less than 0.00001. These results show that a specification that imposes unitary coefficients is readily rejected.

Table 6. Cointegration tests. The tested relation is: $\ln M = \alpha + \beta_1 \ln Y + \beta_2 \ln r + \beta_3 \ln P$. T-statistics in parenthesis

α = -7.462348 β_1 = 1.757152 (12.7671) β_2 = -0.038668 (-3.03321) β_3 = 0.245634 (2.57357)

Hypothesized number of cointegration equations	Eigen value	Trace statistic	5% critical value	Probability
None	0.106086	102.0710	47.85613	0.0000
At most 1	0.023815	28.50340	29.79707	0.0699
At most 2	0.012207	12.69174	15.49471	0.1266
At most 3	0.007040	4.634744	3.841466	0.0313

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 3.

Hypothesized number of cointegration equations	Eigen value	Maximum Eigen value statistic	5% critical value	Probability
None	0.106086	73.56758	27.58434	0.0000
At most 1	0.023815	15.81165	21.13162	0.2360
At most 2	0.012207	8.057001	14.26460	0.3730
At most 3	0.007040	4.634744	3.841466	0.0313

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 3.

More evidence on the fact that equation (5) is the appropriate relation comes from the estimation of equation (6). Table 7 presents the results. These results show many discrepancies. One, the coefficient on the level of the interest rate is statistically significant but has the wrong sign. Two, the coefficient on the log of the price level is also statistically significant but also carries the wrong sign. Three, while the trace statistic finds three cointegration relations, the maximum Eigen value statistic finds only one. These discrepancies are enough to discredit this model and to provide support to the model of equation (5). In turn all equations, besides the unconstrained equation (5), can be considered misspecified because they impose constraints that do not hold statistically. Especially noteworthy is that the income elasticity and the price level elasticity are statistically different from one contrary to what is usually and currently posited. In addition, the fact that equation (5) passes all econometric requirements implies that the money demand relation in the US did not undergo a structural change during the period considered. In other terms the US money demand is stable for the extended period from January 1959 to November 2013.

Table 7. Cointegration tests. The tested relation is: $\ln M = \alpha + \beta_1 \ln Y + \beta_2 r + \beta_3 \ln P$. T-statistics in parenthesis

α = -18.72138 β_1 = 2.933972 (8.74334) β_2 = 92.79540 (5.48385) β_3 = -0.556472 (-2.35828)

Hypothesized number of cointegration equations	Eigen value	Trace statistic	5% critical value	Probability
None	0.131202	124.9337	47.85613	0.0000
At most 1	0.025120	32.53033	29.79707	0.0236
At most 2	0.017567	15.81550	15.49471	0.0447
At most 3	0.006329	4.171662	3.841466	0.0411

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 2.

Hypothesized number of cointegration equations	Eigen value	Maximum Eigen value statistic	5% critical value	Probability
None	0.131202	92.40337	47.58434	0.0000
At most 1	0.025120	16.71482	21.13162	0.1859
At most 2	0.017567	11.64384	14.26460	0.1248
At most 3	0.006329	4.171662	3.841466	0.0411

The actual p-values are retrieved from MacKinnon et al. (1999). The lag length is 2.

By looking on the coefficients on the error-correction variable it is found that only the interest rate variable has a statistically insignificant coefficient. This implies that the money stock, the scale variable, and the price level are endogenous variables while the interest rate is weakly exogenous (Mills and Markellos, 2008).

The next step is to test for the stability and linearity of the error-correction models implied by the two competing cointegration regressions. These two cointegration regressions are: model equation (2) and model equation (5). The first model has a lag length of 2 (see Table 3) and the second a lag length of 3 (see Table 6).The lag lengths are selected by minimizing the Akaike information criterion. The independent variables in the first model are the two lags of the first difference of $\ln M - \ln P - \ln Y$, the current value and the two lags of the change in the level of the short term interest rate, and the lagged cointegration residual (see, for the latter, Table 3). All coefficients are statistically significantly different from zero. Especially noteworthy is the fact that the three coefficients on the interest rate variables are all negative. Hence there is both a negative short run and a negative long run impact of the interest rate on $\ln M - \ln P - \ln Y$ .Non-linearity is tested by finding out if there is one or more calendar break (Bai, 1997; Bai and Perron, 1998). A maximum of five breaks are allowed. Three specifications of the standard errors are considered: robust White standard errors (White, 1980), robust Newey-West HAC standard errors (Newey and West, 1987), and standard errors without any correction for autocorrelation and heteroscedasticity. There is evidence of calendar non-linearity and breaks for all three cases. In the first case there is one break: May 1985. In the second case there are four breaks: January 1969, March 1977, May 1985, and December 1998. In the last case there are three breaks: January 1969, March 1977, and May 1985. Since the number of breaks is different in the three cases this casts doubt on the stability of the short run relation. The conclusion is that the short run model of model equation (2) is not only non-linear but also highly unstable.

Since the lag length is three for the totally unrestricted model equation (5) (see Table 6), then the starting point is an error-correction model with three lags for each variable together with the current values of the growth rate in the short term interest rate, of the growth rate in the scale variable, and of the continuously compounded inflation rate, together with the lagged cointegration residual (ε_{t-1}) retrieved from the relation in Table 6. Hence, omitting a regression error term, this relation is as follows:

$$\Delta(\ln M_t) = \eta + \sum_{i=1}^{3} \alpha_i \Delta(\ln M_{t-i}) + \sum_{i=0}^{3} \beta_i \Delta(\ln Y_{t-i}) + \sum_{i=0}^{3} \delta_i \Delta(\ln r_{t-i}) + \sum_{i=0}^{3} \gamma_i \Delta(\ln P_{t-i}) + \rho \varepsilon_{t-1} \qquad (7)$$

One null hypothesis can then be stated as: $\beta_0 = \beta_1 = \beta_2 = \beta_3 = \delta_1 = \delta_2 = \delta_3 = \gamma_0 = \gamma_2 = \gamma_3 = 0$. This null hypothesis fails to be rejected with and actual p-value of 0.1923 when applying the likelihood ratio test statistic. The remaining coefficients are all statistically significantly different from zero with a lowest absolute t-statistic of 2.781113. The coefficient δ_0 is negative as expected with a t-statistic of -4.185644. The coefficient on the lagged regression residual ρ is also negative as expected with a t-statistic of -4.606926. What is noteworthy is the negative coefficient on the first lagged value of the inflation rate γ_1 which has a t-statistic of -5.789261. It seems that in the short run the inflation rate is negatively related to the growth in the nominal money stock, while in the long run the price level is positively related to the log level of the money stock. This is not totally unrealistic. Non-linearity is tested by finding out if there is a one or more calendar break (Bai, 1997; Bai and Perron, 1998). A maximum of five breaks are allowed. Three specifications of the standard errors are considered as above: robust White standard errors, robust Newey-West HAC standard errors, and standard errors without any correction for autocorrelation and heteroscedasticity. In the first case and the third case there are the same four breaks: February 1970, September 1982, February 1993, and December 2002. In the second case no breaks are selected. Hence there is some evidence for non-linearity but even this evidence of non-linearity supports stability of the relation since the same break points are selected. All this confirms that relation equation (5) is the best specification even if there is some support for non-linearity. Therefore the evidence for a liquidity trap is strong.

5. **Conclusion**

The purpose of this paper is to estimate a long run money demand function for the United States using monthly data from January 1959 till November 2013. The paper tries to find out whether the interest rate variable should enter in a log functional form or not. The two functional forms imply different theoretical underpinnings. In addition constrained and unconstrained money demand functions are tested for stability over the whole period. The usual constraints are unitary coefficients on the scale variable and on the price level variable. When these coefficients are constrained to be unitary the best specification is obtained by including the interest rate variable as is. When all constraints are relaxed the best specification is when the interest rate variable enters in logs. There is evidence that the imposed constraints do not hold statistically. The main conclusion is therefore that a complete log-log model of money demand is the most appropriate, although there is some evidence for non-linearity, represented by calendar breaks. This implies that, at very low interest rates, there is a Keynesian liquidity trap wherein monetary policy becomes totally ineffective.

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