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Comparing the Effectiveness of Inquiry-Based Learning and Direct Instruction on Enhancing Mathematical Thinking in Secondary School Students

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ABSTRACT

This study aims to investigate the effect of inquiry-based learning and direct instruction in enhancing mathematical thinking abilities among secondary school students to examine students' attitudes towards inquiry-based learning as a method for improving mathematical thinking abilities. This study adopted the quantitative research method. Eighty-six questionnaires were issued, and 86 were valid, with a validity of 83.5%. The mathematical thinking ability test consists of pre-test and post-test items. There are 43 students from Class 1- control group and 43 students from Class 2- experiment group, Grade 3. This study finds that the achievements of inquiry-based learning are better than direct instruction in improving secondary school students' mathematical thinking abilities; students who learn with inquiry-based learning to improve secondary school students' mathematical thinking abilities have a high level of satisfaction. Based on the analysis results, the following suggestions have been put forward. Teachers should cultivate students' confidence and abilities by guiding more active discussions and cooperation. Teachers should cultivate students' patience and strategic awareness in problem-solving. This teaching method's success relies on teacher guidance, an open classroom atmosphere, and active interaction among students.

Key words: Inquiry-Based Learning Mathematical Thinking Abilities Secondary School Students

INTRODUCTION

In recent years, educational and learning theories have evolved alongside the waves of educational reform, prompting schools and teachers to reevaluate long-standing teaching and talent development (Gopinathan et al., 2022). With the deepening reform of mathematics education in China, inquiry-based learning in mathematics has garnered increasing attention. Since China introduced new curriculum reform standards, new demands have been placed on teachers, advocating for active interaction and mutual development between teachers and students during the teaching process. The Chinese Ministry of Education "General Middle School Mathematics Curriculum Standards" (2017 edition, revised in 2020) clearly states in its section on the nature of the curriculum that "mathematics plays an irreplaceable role in shaping rational thinking, scientific spirit, and promoting the intellectual development of individuals." (Chen et al., 2020). Mathematics is closely related to rationality; mathematical thinking abilities are an indispensable form of thinking for everyone in modern society.

The traditional, singular approach to mathematics instruction, which progresses from "observation" to "analysis" to "expression," no longer meets the requirements for cultivating students' mathematical core literacy. Mathematics educators must seek breakthroughs, adopting more diverse teaching methods emphasizing process and skill development. For students, mathematics should not be merely a process of "input" but also a process of "output." (Cui et al., 2019). In China's current stage of mathematics education, the teaching approach is predominantly exam-oriented, focusing on improving students' test scores. Teachers typically lead the instruction, while students' learning behavior is reduced to simple imitation. This results in a dull and passive classroom atmosphere, leading to low teaching efficiency (Cui et al., 2019). Therefore, every mathematics educator must consider establishing an effective learning system. Inquiry-based learning within modern education offers a new approach to learning, allowing students to re-explore, understand, and organize mathematical knowledge. Through inquiry-based learning, students gain a deeper understanding of mathematical concepts and construct cognitive structures (Gopinathan et al., 2022; Guo et al., 2018). This process also allows mathematics teachers to gain deeper insights into students' learning, providing a new, equitable platform for dialogue between teachers and students.

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Research Questions

1. How does inquiry-based learning compare to direct instruction in enhancing mathematical thinking abilities among secondary school students?
2. What are the students' attitudes towards inquiry-based learning as a method for enhancing mathematical thinking abilities in secondary school?

Research Hypotheses

1. The achievements of inquiry-based learning are better than direct instruction in improving secondary school students' mathematical thinking abilities.
2. Students who learn with inquiry-based learning to improve secondary school students' mathematical thinking abilities are delighted.

Research Objectives

1. To investigate the effect of inquiry-based learning and direct instruction in enhancing mathematical thinking abilities among secondary school students.
2. To examine students' attitudes towards inquiry-based learning as a method for improving mathematical thinking abilities in secondary school.

LITERATURE REVIEW

Constructivist Theory

The roots of constructivist theory can be traced back to Piaget's theory of cognitive development in children (Schmidt, 1987). According to Piaget, children's cognitive development is achieved through continuous interaction with the environment and the absorption and integration of new information during this process (Schmidt, 1987). Children's thinking is dynamic and constantly evolving; they gradually build more complex cognitive structures by adapting to new environments and information. Constructivists draw from this idea, asserting that knowledge acquisition is not the result of passive reception but an active construction (Zander, 2010). When faced with new information, learners first attempt to assimilate it into their existing cognitive structures -a process known as assimilation. However, when new information does not fully align with existing knowledge structures, learners must adjust or expand cognitive structures to better integrate the knowledge (Anyanwu et al., 2024). This adjustment process is referred to as accommodation (Tay, 2016).

Constructivism highlights the contextual nature of learning, asserting that knowledge acquisition must be connected to actual social and cultural contexts. This means that knowledge is not merely an abstract concept or theory but is closely related to concrete practices and applications (Watson, 2000). Learners can better understand the meaning and value of knowledge and transform it into practical skills by applying it in real-life situations. Constructivist theory reveals that the acquisition and understanding of knowledge is an active, dynamic, and contextualized process (Deibl et al., 2018). Learners integrate and adjust new information with existing knowledge structures through interaction with the environment, thereby achieving cognitive development. This theory has profoundly influenced educational practice, from teacher-centered to student-centered learning models emphasizing learners' active participation in knowledge construction (Fong et al., 2016). Constructive learning involves learners continuously comparing new and existing knowledge to form and adjust their cognitive structures (Chen & Anyanwu, 2025). Based on constructivist theory, learners must build knowledge structures (Deibl et al., 2018). In the process of constructive learning, the teacher acts as a guide for student learning, while students are the main subjects of learning. Teachers lead students in constructing their knowledge systems, and students actively select, process, and interpret external knowledge from teachers or other sources, then recode and reinterpret the newly acquired knowledge, integrating it into their personal knowledge system to form their understanding (Deibl et al., 2018).

Cognitive Assimilation Theory

Cognitive psychology is a branch of psychology that studies cognitive processes from an information-processing perspective (Tian et al., 2019). Cognitive psychologists infer unobservable mental processes based on observable phenomena and have provided unique educational insights. Knowledge is considered not to be predetermined within the cognitive subject but is primarily acquired through external integration. When children incorporate relevant information from the external environment into their existing cognitive structures (also known as "schemas"), this process is referred to as "assimilation." If new knowledge cannot be assimilated, it triggers "accommodation," a process where changes in the external environment lead to modifying and reorganizing existing cognitive structures when they can no longer assimilate the new information (Nang et al., 2022). Assimilation and accommodation are the two fundamental processes by which students perceive and understand the world. Assimilation leads to expanding cognitive structures (i.e., schema expansion), while accommodation changes the nature of cognitive structures (i.e., schema modification). Children's cognitive structures are gradually constructed through the balance between assimilation and accommodation in interaction with their environment (Shogren et al., 2006).

The function of teaching is to facilitate student development, and authentic tasks are more likely to stimulate student learning. Cognitive assimilation theory places great importance on students' learning tasks (Ritter et al., 2017). Through learning tasks, students become aware that their current level of development is insufficient to complete the tasks, requiring guidance from teachers or assistance to promote development (Nang et al., 2022). When learners recognize their deficiencies in knowledge, it fosters active thinking and a willingness to seek help, making learning more proactive and inquiry-based (Capps& Crawford,2013). Scholars of cognitive assimilation theory believe that one of the main functions of school education is to stimulate students' interest in learning (Darlington, 2014) by creating cognitive conflicts or incomplete cognitive systems, motivating learners to work towards resolving them. The stimuli that cause cognitive conflict originate from learning tasks (Fong et al., 2016). Students can engage in classroom learning in a manner that closely resembles real-life situations, incorporating the knowledge presented by the teacher into their cognitive systems through active thinking, hands-on activities, and the summarization and organization of information (Fong et al., 2016; Tian et al., 2019). In this process, students personalize their interpretation of knowledge while experiencing the practical connection between learning and life.

Related Studies

Inquiry-Based Learning (IBL) was advocated by Professor Schwab of the University of Chicago in the 1950s as part of the "Modernization of Education" movement (Rasi, 2015). Aparicio-Ting, Slater, and Kurz (2019) focus on a phased approach to Inquiry-Based Learning (IBL) as a curriculum driver. Oliver et al. (2019) explored the perceptions and practical application of IBL among science teachers in the UK and Spain. Their research indicated that although IBL is highly recommended in education, its actual use in science education in the UK and Spain is relatively limited (Bao et al.). Inquiry-based learning typically involves selecting and defining research topics from academic disciplines or real-life scenarios, creating contexts for students (Aparicio-Ting et al., 2019). This approach aims to enhance students' knowledge, skills, emotional attitudes, and spirit of inquiry and innovation. Aditomo, Goodyear, Bliuc, and Ellis (2013) explored the principal forms, educational objectives, and disciplinary variations of Inquiry-Based Learning (IBL) in higher education. The research found that IBL can effectively stimulate students' curiosity and cultivate their spirit of inquiry. Ibrohim et al. (2020) conducted a study on the implementation of Inquiry-Based Learning (IBL) to enhance students' understanding of the nature of science (NOS).

Inquiry-Based Learning (IBL) is an educational approach that encourages students to actively engage in learning by applying their existing knowledge and life experiences to tackle challenging problems or tasks (JingXuan, 2024). By guiding students through independent learning and collaborative discussions, IBL deepens their understanding of scientific and mathematical concepts, enhances their inquiry skills, and promotes innovation (Buckner & Kim, 2013). Inquiry-based learning is characterized by openness, involving the interplay and disorder between learning content and individuals' experiences and knowledge. This disorder stimulates students' interest in organizing and making sense of information, driving their desire to acquire knowledge (Aditomo et al., 2013; Aparicio-Ting et al., 2019; Rasi, 2015). Effective inquiry-based learning requires active regulation by teachers to manage students' problem-solving abilities, possible inquiry directions, and emotional engagement (Singh, 2024).

Mathematical thinking ability encompasses mathematical computation skills, spatial imagination skills, and logical thinking skills, with thinking considered the core of mathematical thinking (Dewi et al., 2019). According to the "General Standard for Secondary School Mathematics Curriculum" (2017 edition, revised in 2020), students' mathematical thinking abilities include intuitive perception, observation, induction, analogy, spatial imagination, abstraction (Heleni & Zulkarnain, 2018), symbolic representation, computation, data processing, deductive proof, and reflection and construction (Chen et al., 2020). Mathematical thinking ability is described from two distinct perspectives. It comprises twelve specific abilities: discovering attributes, mathematical variation, identifying similarities, mathematical reasoning, mathematical transformation, intuitive thinking, conceptual generalization, generalization of mathematical principles, adaptive generalization, discovering relationships, pattern recognition, and using cognitive blocks (Dewi et al., 2019). These abilities are categorized into conventional mathematical thinking abilities and innovative abilities. Conventional mathematical thinking includes ten aspects: numerical and geometric intuition and judgment, data collection and analysis, geometric intuition and spatial imagination, mathematical representation and modeling, and mathematical operations and transformations (Mustafa et al., 2019).

RESEARCH METHODOLOGY

This study adopts a quantitative research design to investigate the impact of inquiry-based learning on secondary school students' mathematical thinking abilities. This study will design a survey questionnaire, a learning plan, and a mathematical thinking test. The research aims to explore the effectiveness of this learning approach in enhancing mathematical representational, logical, and intuitive thinking. The study will target secondary school students, randomly selecting classes and dividing them into experimental and control groups. The experiment group will receive mathematics instruction centered on inquiry-based learning, while the control group will continue with traditional direct instruction. Inquiry-based learning involves several key steps: pose questions related to the topic, encourage student independent exploration, analyze and explain answers to questions, guide students to summarize, reflect, and evaluate the exploratory learning process, sharing and communicate findings. Each step guides students through active participation, collaboration, and critical thinking to deepen their understanding of mathematical concepts. At the beginning of the study, all participants will undergo a series of mathematical tests to assess their initial levels of mathematical representational thinking, logical thinking, and intuitive thinking.

The questionnaire will be divided into two sections. The first section will gather basic demographic information about the sample, including gender, age, and other demographic characteristics. The second section will consist of measurement items based on the components of inquiry-based learning. It will measure students' satisfaction with various aspects of inquiry-based learning, with five items designed for each process, totaling 30 items. The questionnaire will use a Likert five-point scale for responses. The cronbach's alpha equal 0.90. The learning plan will be designed according to the steps of inquiry-based learning. The plan will include specific content for developing students' mathematical thinking, teaching practices, and implementation steps. The mathematical thinking test will be divided into two parts. The first part will assess students' mathematical thinking abilities before they undergo inquiry-based learning to determine any pre-existing differences. The second part will evaluate students' mathematical thinking abilities after they have experienced both inquiry-based and direct instruction, measuring the differences in their mathematical thinking abilities. The IOC equal 1.00.

FINDINGS

Demographic Characteristics

The students in Class 1 and Class 2, Grade 3, of Guang Ming Middle School demonstrate a relatively balanced distribution in demographic variables, yet they also exhibit some notable differences and characteristics. For gender, boys and girls account for 50% each, with 43 boys and 43 girls. This indicates a balanced gender ratio within the classes, avoiding a situation where one gender is overly dominant, and provides equal observation conditions for teaching research, especially when analyzing gender differences. There are significant differences in age distribution. Thirty-eight students (44.2%) are under 14 years old, belonging to the younger student group, while 18 students (20.9%) are 14 years old, and another 18 (20.9%) are over 15 years old, accounting for a proportion. Additionally, 12 students (14%) are 15 years old. For academic performance (GPA), most students' GPAs are concentrated in the range of 2.6-3.0, with 40 students (46.5%) falling into this relatively moderate performance bracket. There are 25 students (29.1%) with GPA between 2.1 and 2.5; students have moderate academic performance. Only a few students have GPAs higher than 3.5, with eight students (9.3%), and a similarly small number have GPAs below 2.0, totaling ten students (11.6%). Few students perform excellently or poorly. Moreover, scores are concentrated in the middle range (Table 1).

Investigation of the Impact of Inquiry-Based Teaching on Mathematical Thinking Ability

The subjects of this study are the students in Class 1 and Class 2, Grade 3 of Guang Ming Middle School. Students in Grade 3 of this school learn mathematics through two methods: inquiry-based teaching and direct learning. Class 1, Grade 3 will serve as the experimental group (inquiry-based teaching), while Class 2, Grade 3 will serve as the control group (direct learning). Tests will be conducted twice, once before the start of the study (pre-test) and once after its conclusion (post-test). The pre-test aims to assess the students' baseline mathematical thinking ability before they receive different teaching methods, while the post-test assesses changes in their ability after receiving instruction. The test content should align with the curriculum outline and cover questions related to mathematical representational thinking, logical thinking, and intuitive thinking.

The achievements of inquiry-based learning are better than direct instruction in improving secondary school students' mathematical thinking abilities.

Based on the results in Table 2, we fail to reject the null hypothesis. This means the test variance of class inquiry-based learning equals the variance of class direct instruction. So, after learning, use the t-test to equal the variance.

Test average the achievement of inquiry-based average the achievement of secondary school students' mathematical thinking abilities.

Based on the results in Table 3, we reject the null hypotheses that the achievements of inquiry-based learning equal that of direct instruction. So, inquiry-based learning is better than direct instruction in improving secondary school students' mathematical thinking abilities.

Examine Students' Attitude Towards Inquiry-Based Learning as A Method for Improving Mathematical Thinking Abilities in Secondary School

The data collected from the survey questionnaire is reliable and valid. The research data present descriptive statistics on students' attitudes toward inquiry-based teaching, specifically categorized into six dimensions: questioning, investigation and research, analysis and interpretation, conclusion

Table 1. Descriptive characteristics

Options	Frequency	Percentage
Gender		
Male	43	50.0
Female	43	50.0
Age		
Under 14	38	44.2
14	18	20.9
15	12	14.0
Over 15	18	20.9
Under 2	10	11.6
GPA		
2.1-2.5	25	29.1
2.6-3.0	40	46.5
3.1-3.5	3	3.5
Over 3.5	8	9.3
Total	81	100.0

Table 2. Compare variance of class inquiry-based and variance of class direct instruction (before learning)

Class	df	*M*	*Var*	F	*p*-value
Inquiry-Based	42	7.67	1.35	0.92	0.40
Direct Instruction	42	7.67	1.46		

Table 3. Compare average achievement of inquiry-based learning and average achievement of direct instruction

Class	n	*M*	*Var*	df	t
Inquiry-Based	43	13.84	0.95	84	6.50*
Direct Instruction	43	11.79	3.31		

$t_{0.05.84}$=1.66 $t_{compute}$=6.50 > $t_{0.05.84}$=1.66

Table 4. Show items, mean, standard deviation, and interpret of satisfaction

Dimension	*M*	*SD*	Interpret
Questioning	3.45	1.166	Moderation
Investigation and Research	3.60	1.153	High
Analysis and Interpretation	3.50	1.195	Moderation
Conclusion Formation	3.69	1.137	High
Reflection and Evaluation	3.83	1.023	High
Sharing and Communication	3.61	1.177	High
Total	3.61	1.15	High

formation, reflection and evaluation, and sharing and communication. Each dimension contains multiple items, presenting the mean (*M*) and standard deviation (*SD*) for each item. See Table 4 for details.

In the questioning dimension, the *M* range is from 3.36 to 3.55, with an overall *M* of 3.45, and the *SD* are between 1.07 and 1.27. These indicate that students' performance in the questioning stage is slightly above average, but the distribution of scores is relatively dispersed, suggesting significant differences in students' abilities or attitudes.

In the investigation and research dimension, students have a slightly higher *M*, with an overall *M* of 3.60 and a *M* range of 3.51 to 3.71. The *SD* are ranging from 0.98 to 1.31. These suggest that students have a higher level of engagement in investigation and research, compared to the Questioning dimension, there is less variation among students.

The *M* for the analysis and interpretation dimension is 3.50, with item *M* ranging from 3.30 to 3.63 and *SD* between 1.11 and 1.31. Although the mean for this dimension is slightly lower, the *SD* indicate some variation in students' analytical and interpretive abilities. These may suggest that some students perform more prominently at this stage.

The *M* for the conclusion formation dimension is 3.69, with a *M* range of 3.50 to 3.81 and *SD* between 1.03 and 1.31. The higher *M* indicates that students perform relatively well in the conclusion formation stage, and the *SD* show that there is still some variability among students.

The reflection and evaluation dimension has the highest *M* of 3.83, with individual item means ranging from 3.71 to 4.06 and relatively small *SD*, ranging from a low of 0.82 to a high of 1.21. These indicate that students have a strong sense of reflection and evaluation of their learning process and show positive performance. The data for this dimension suggest that most students have high engagement and relatively balanced performance at this stage.

The overall *M* for the sharing and communication dimension is 3.61, with a *M* range of 3.43 to 3.71 and *SD* between 1.12 and 1.25. The *M* is lower, suggesting that students' performance in sharing and communication may not be as strong as in other dimensions.

Overall, students have a generally positive attitude towards inquiry-based teaching, with the most prominent performance in the reflection and evaluation and conclusion formation dimensions, reflecting their activity and confidence in the later stages of the learning process (i.e., conclusion drawing and reflection/evaluation stages). However, the means for the questioning and communication dimensions are relatively lower, and the standard deviations are high. These suggest that teachers should pay more attention to these aspects in their teaching, potentially guiding students to ask better questions and share and communicate more effectively.

CONCLUSION

The Achievements of Inquiry-Based Learning Are Better Than Direct Instruction in Improving Secondary School Students' Mathematical Thinking Abilities

Before implementing inquiry-based teaching, a data test was conducted among the students of Class 1 and Class 2, Grade 3 at Guang Ming Middle School, and 86 were valid. An independent sample t-test was performed on the student's scores in the two classes. The research results showed a F-value, df (degrees of freedom) of 84, and a significance value (Sig. (1-tailed)) of 0.40. Since the significance value was 0.40, much greater than 0.05, it indicated that there was no statistically significant difference in the test scores between the two classes. After implementing inquiry-based teaching, another data test was conducted among the students of these two classes, and the t-test was conducted. Based on the results of the t-test, it could be concluded that the average scores of the experimental group (inquiry-based teaching) in the post-test were significantly higher than those of the control group (direct instruction), indicating that inquiry-based teaching had a significant positive impact on students' test scores. Therefore, the research results suggest that the hypothesis that inquiry-based learning is more effective than direct instruction in improving secondary school students' mathematical thinking abilities holds.

Before the implementation of inquiry-based teaching, the test scores of Class 1 and Class 2, Grade 3 at Guang Ming Middle School showed no significant difference. This meant that before the experiment began, the students in the two classes had a balanced mathematical foundation and ability, providing a good comparison basis for the subsequent teaching experiment. After the implementation of inquiry-based teaching, the average scores of the experimental group were significantly higher than those of the control group in the post-test. This result demonstrated the effectiveness of inquiry-based teaching in improving students' mathematics scores and further illustrated its enhancement of students' mathematical thinking abilities. Inquiry-based learning emphasizes the process of students asking questions, investigating, analyzing, and forming conclusions independently, which aligns well with the logical reasoning, critical thinking, and abstract thinking abilities required for mathematical thinking.

In contrast, direct instruction typically focuses more on teachers' explanations and students' memorization and imitation, which can improve students' mastery of basic concepts and skills in the short term but may be limited to fostering deep mathematical thinking. In inquiry-based teaching, students actively explore problems and learn to view issues from different perspectives, thereby developing more complex ways of thinking. This process is not just about finding the correct answer but about understanding the mathematical structures behind the problems and how to apply existing knowledge to solve new problems. Inquiry-based teaching also provides students with more opportunities for reflection and evaluation, which is particularly important for mathematical thinking. Students can examine processes, identify problems, and revise their ideas. The cultivation of this ability is undoubtedly crucial for mathematical thinking. Conversely, direct instruction rarely provides students with such opportunities, and students often judge right and wrong based on the standard answers given by teachers, lacking the space for independent examination and evaluation.

Thus, by giving students more autonomy and encouraging them to continuously ask questions, reflect, and improve during the learning process, inquiry-based teaching enables students to make significant progress in mathematical thinking. Compared with direct instruction, it more effectively promotes the development of students' logical reasoning, problem-solving, and critical thinking abilities, enabling them to solve problems and understand the mathematical principles behind them. This enhancement of abilities is reflected in academic performance and impacts student's future learning and life.

Students Who Learn with Inquiry-Based Learning to Improve Secondary School Students' Mathematical Thinking Abilities Have a High Level of Satisfaction

The research data presents descriptive statistics on students' attitudes towards inquiry-based teaching, specifically categorized into six dimensions: posing questions, investigation and research, analysis and interpretation, forming conclusions, reflection and evaluation, and sharing and communication. Overall, students hold a positive attitude toward inquiry-based teaching, with outstanding performance in the dimensions and forming conclusions. The results reflect that students are more active and confident in the later stages of the learning process, namely drawing conclusions and reflecting/evaluating. However, the mean values for posing questions and sharing are low, indicating significant individual differences among students.

These results suggest that inquiry-based teaching is a method to enhance mathematical thinking abilities. Therefore, based on the research findings, the hypothesis that student satisfaction increases through inquiry-based learning holds. In an open learning environment, differences in students' thinking abilities and expressive skills may be magnified, with some students finding their pace in inquiry-based teaching may require less guidance and support. Thus, although students' attitudes towards inquiry-based teaching are overall positive, this positivity is not evenly distributed. To address the challenges in posing questions and sharing/communicating, incorporating more guiding questions or adopting group cooperation may help those more passive students better integrate into the inquiry-based learning process. These results suggest that when implementing inquiry-based teaching, teachers need to flexibly adjust their teaching methods according to the needs of different students to ensure that each student can gain a meaningful learning experience across all dimensions.

These advantages of inquiry-based teaching enable students to transcend their understanding of specific mathematical problems and cultivate broader mathematical literacy. It provides students with a space for continuous growth, enabling them to cope with current learning challeges and lay a solid foundation for future mathematical studies. This teaching method's success relies on teacher guidance, and open classroom atmosphere, and active interaction among students. Through these practical aspects, inquiry-based teaching makes students more autonomous and confident in mathematics learning and provides long-term assistance for their intellectual development.

DISCUSSION

Part 1: How does Inquiry-Based Learning Compare to Direct Instruction in Enhancing Mathematical Thinking Abilities Among Secondary School Students?

Inquiry-based teaching promotes the in-depth development of students' mathematical thinking in various ways. It emphasizes students' ability to pose questions, explore different problem-solving strategies, and construct their mathematical understanding during the learning process. This viewpoint aligns with the research by Oliver et al. (2019): Ibrohim et al. (2020). Compared to the knowledge transmission approach of direct instruction, inquiry-based teaching stimulates students' active thinking and logical reasoning abilities. Inquiry-based teaching encourages students to seek multiple solutions to gain a deeper understanding of the essence of mathematical concepts, which is reflected in the post-test results—students in the experimental group performed significantly better than those who only received direct instruction.

Inquiry-based teaching can promote the in-depth development of students' mathematical thinking because it emphasizes students playing a more active role in the learning process. Students engage in learning by posing questions and exploring different problem-solving strategies. This method encourages students to think about problems from multiple angles, try various possible solutions, and ultimately construct their unique understanding of mathematical concepts in this process. These conclusions are consistent with the research findings of Nang et al. (2022). Inquiry-based teaching provides students with an open learning environment where they can grow through trial and error. Each attempt to solve a problem, even if it leads to helping students gain a deeper understanding of the principles behind mathematics, thereby promoting the development of their abstract thinking abilities. This cultivation of abstract thinking is not achieved by imparting fixed problem-solving steps but through repeated exploration and comparison of different solutions. Collaborative learning in inquiry-based teaching also enhances students' self-confidence and team collaboration abilities. When exploring complex problems, students can discuss with each other, share their ideas and thoughts, and gain new inspiration from these exchanges. This viewpoint is consistent with the research by Koufaris (2022): Chen et al. (2020). This collision of collective wisdom enhances students' understanding of mathematical problems and helps them more confidently face unknown challenges in future learning.

Therefore, inquiry-based teaching stimulates students' curiosity, learning, and participation in reconstructing knowledge. This method helps students progress in mathematical thinking and enhances their ability to solve practical problems. In this process, students learn how to think, question, and collaborate, thereby laying a solid foundation for their future academic development.

Part 2: What are the Students' Attitudes Towards Inquiry-based Learning to Enhance Mathematical Thinking Abilities in Secondary School?

Research data presents descriptive statistics on students' attitudes toward inquiry-based teaching, categorized into six dimensions: questioning, investigation and research, analysis and interpretation, conclusion formation, reflection and evaluation, and sharing and communication. In the questioning dimension, the *M* is 3.45, which means "agree" (interpreted as high). The *M* for the investigation and research dimension is 3.60, indicating "agree" (interpreted as high). The *M* for the analysis and interpretation dimension is 3.50, which means "moderate" (interpreted as moderate). The *M* for the conclusion formation dimension is 3.69, signifying "agree" (interpreted as high). The reflection and evaluation dimension has a mean of 3.83, meaning "agree" (interpreted as high). Overall, students hold a generally positive attitude towards inquiry-based teaching, with the most prominent performance observed in the reflection, evaluation, and conclusion formation dimensions, reflecting students' activity and confidence in the later stages of the learning process (i.e., conclusion drawing and reflection/evaluation stages). However, the means for the questioning, sharing, and communication dimensions are relatively lower, with larger standard deviations, indicating significant individual differences among students in these areas.

From the research findings, students' attitude toward inquiry-based teaching exhibits a positive trend, particularly in the crucial stages of reflection, evaluation, and conclusion formation. Students demonstrate considerable confidence in the later stages of the learning process when continuously reflecting on and evaluating their learned content. This confidence is their ability to draw clear mathematical conclusions and summarize them. In these stages, students' average attitude scores are higher, showing that they have accepted the challenges posed by inquiry-based teaching and gained positive feedback, gradually establishing a sense of control and satisfaction in their learning. This later-stage autonomy enhances their mathematical thinking abilities and deepens their consideration of complex problems. This viewpoint aligns with Sholehawati and Wahyudin's (2019) research.

Students' questioning, sharing, and communication performance is relatively less prominent. The lower score in the questioning dimension suggests that students may lack sufficient confidence to actively ask questions or express their doubts when faced with unfamiliar or complex mathematical problems. Such situations may stem from various reasons, such as students' insufficient understanding of mathematical problems or unfamiliarity with this open-ended way of thinking in the classroom. In an inquiry-oriented classroom, asking questions is the first step in learning, and students' differences in this aspect indicate that some students may face challenges in adapting to this teaching method. This viewpoint is consistent with the research by Nurmanita et al. (2019).

The sharing and communication aspect is noteworthy. Inquiry-based teaching emphasizes cooperative learning and group discussions, but the data shows that students' enthusiasm in this area is relatively low, with significant variation. This may reflect students' different levels of acceptance of classroom interaction, with some students being more adept at independent thinking or feeling less comfortable when sharing. This phenomenon suggests that when promoting inquiry-based teaching, teachers may need to consider encouraging students to be more active in communication during cooperative learning and establish a more inclusive classroom atmosphere so that all students can feel safe and supported in the discussion environment. This viewpoint aligns with the research by Susilawati et al. (2019).

The results demonstrate students' positive attitudes towards inquiry-based teaching, especially their outstanding performance in the later reflection and summary stages. This indicates that inquiry-based teaching helps students deeply understand mathematical problems and enhances their thinking flexibility and self-evaluation abilities. In this teaching mode, students have to reflect on their learning paths, evaluate their thinking processes, and adjust based on these reflections, undoubtedly laying a solid foundation for their mathematical learning.

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