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Interplay between Language and Mathematics Comprehension/Learning: A Direction Dependence Analysis

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ABSTRACT

The present study aims to inspect the interplay between language (Turkish) comprehension/learning (LCL) and mathematics comprehension/learning (MCL). We utilized a mixed-methods exploratory research design to understand the mechanism between LCL and MCL. First, we used an analytic rubric to score a researcher-developed achievement battery consisting of seven open-ended language (Turkish) and seven open-ended mathematics new generation questions. Although these were multiple-choice questions, participants were solicited to write a detailed response on why they endorsed a particular choice. In the quantitative section, open-ended responses for each question were rated by two independent subject experts. The average of these two ratings was used to derive factor scores for LCL and MCL. Factor scores were used in direction dependence analysis to determine the magnitude and likely direction of the effect between LCL and MCL. In the qualitative section, we conducted unstructured interviews with the selected participants to get more detailed responses regarding their decision steps. Content analysis was performed on the transcribed voice-recordings. Results revealed that, overall, it is more likely that higher scores on LCL predicted higher scores on MCL. However, the strength and direction of the prediction varied in low, medium, and high-achieving groups. MCL and LCL did not seem related in the low-achieving group; higher scores on MCL predicted lower scores on LCL in the medium-achieving group, whereas higher scores in LCL predicted higher scores in MCL in the high achieving group. Qualitative results support quantitative findings. It seems high achieving students dominate the LCL and MCL relationship. Longitudinal studies (e.g., cross-lagged panel design) are needed for more conclusive results.

Key words: Language, Mathematics, Comprehension, Learning

INTRODUCTION

Language, which encompasses all aspects of human life and reflects ideas in concrete form (Wittgenstein, 2008), and mathematics, representing ideas in symbols (O'Halloran, 2015), are two interrelated disciplines. The review of the definitions proposed in previous studies on language and mathematics demonstrated that these definitions included the same content. Language is an advanced system that leads to an understanding among individuals and mediates the expression of emotions and ideas, as well as signs, sounds, shapes, strings, symbols, codes, etc. based on certain rules (Aksan, 2009; Barre et al., 2011; Catts et al., 2006; Ergin, 2009; Gencan, 2007; Hengirmen, 2007; Vardar, 2007). Mathematics is a universal language (Adoniou & Qing 2014; Waller & Flood, 2016) that allows the communication between individuals through terms, concepts, symbols and grammar (Cirillo et al., 2010). Thus, language and mathematics are advanced systems that include a realm of symbols, images and sequences with specific rules and terms. Mathematics is not only independent from language (Ní Ríordáin & O'Donoghue, 2009) but also a multi-semiotic science (O'Halloran, 2015). Furthermore, it was argued that language is a magical system (Benjamin, 2011; Hauser et al., 2014) with a structure similar to mathematics (Harris, 1968). These ideas demonstrated that language and mathematics intersect on several areas (Nesher & Katriel, 1986).

Language is a system of rules and codes that allow communication (Harley, 1995). Humans have the innate language skills. These skills symbolize a cognitive potential (ability) for systematic lingual codes (Chomsky, 1972). Thus, mathematics and language share certain concepts such as thought, perception, symbol, code, rule, knowledge, skill, learning, comprehension, etc. The ability of language that is based on human feelings, thoughts and curiosity (Barre et al., 2011; Catts et al., 2006) to produce infinite sentences based on certain structures (Chomsky, 1992) is consistent

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with the mathematical concept of infinite series of numbers (Kaufmann, 1978; Russell, 1920). The idea that individuals (students) could obtain infinite numbers from certain numbers in mathematics (Pehkonen et al., 2006) and the view that infinite sentences could be produced in a given language based on certain rules (Chomsky, 1965, 1972) are similar. Thus, language and mathematics are two correlated disciplines having unique or common symbols.

Goodstein (1970) discussed lingual structures based on mathematical constructs such as set theory, mathematical logic, quantitative theories and lingual variables. They argued that language has a mathematical structure. Furthermore, Harris (1968) reported that language could be explained by mathematical formulas via the lingual properties of mathematical equations. Plato discussed the doctrine of ideas based on the doctrine of forms. Plato's cosmology included the idea that reality is formed by mathematical shapes and forms, not unassociated data (Peters, 2004; Vernant & Vidal-Naquet, 2012). Furthermore, Piaget's (1959) "egocentric", Vygotsky's (1986) "inner speech" and "external speech" concepts are ultimately based on comprehension and concretization (signification). Therefore, language and mathematics are connected within the framework of thinking and comprehension.

The common point among those who described language as a system of symbols (Saussure, 1916; Sapir, 1921; Francis, 1958; Finochiaro, 1974) and those who emphasized the cognitive aspects of language (Chomsky, 1957, 1965, 1972, 1995; Millikan, 2005; Carruthers, 2002) is the presence of ideas, symbols, codes, and rules. The elements such as thoughts, sounds, signs, etc. are reflected to the outside world through language (Asoulin, 2016). These indicators (sounds, signs, etc.) are more syntactic and semantic in the language (Asoulin, 2016). This occurs mainly through the thought system and productivity in the language (Boeckx & Piattelli-Palmarini, 2005; Brattico & Lassi, 2009; Chomsky, 1965, 1972). The structure of mathematical semantic correlations depends on language and mathematical symbols (Ferah, 2006, p. 119). Therefore, in order to comprehend success and failure in mathematics, it is necessary to understand the relationship between language and mathematics (Wood 2003). Linguistic development also means the thought development. Linguistic and thought development assists the development of the individual's metacognitive skills such as comprehension, reasoning, interpretation-evaluation, analysis and synthesis. Thus, language and thought complement each other, and one cannot be complete without the other. According to Adsoy, (2020), Coşkun (2014) and Ferah, (2006), since language and thought are two complementary concepts, it is not possible for one to improve without the other. Actualization of a thought into action and language is possible by the organization (integration) of words in imagination (Piaget, 1959, 1972). Chomsky's (1957, 1968, 1972) "innateness of language", Piaget's (1959) "egocentric" and Vygotsky's (1986) "inner speech" and "external speech" concepts were associated with language skills and development. Due to the correlations between language, mathematics and metacognitive skills, it could be suggested that language development affects metacognitive skills (Souviney, 1983, Perez & Alieto, 2018). Thus, language, thought, and mathematics should be considered as a whole.

Aristoteles (1947) categorized thought as theoretical, applied and creative. This view of Aristotle (1947) was related to Kant's (2016, p. 12-14) view that knowledge was defined in the mind and is the combination of cognitive representations. Studies in neuroscience (Demeyere et al., 2012; Sophian & Crosby, 2008) demonstrated that mathematics is closely associated with thought, language and mind (Nufus & Ariawan, 2019; Norton & Deater-Deckard, 2014). Due to its cognitive property, the language could produce infinite sentences (Chomsky, 1957, 1965) and the brain (mind, language skills) is constantly renews and develops itself (Quartz & Seinowski, 1997), and mathematics produces endless propositions that include various numbers (Kaufmann, 1978; Russell, 2019). These properties demonstrate that language and mathematics are two disciplines that affect one another based on cognition. Mathematics, similar to the language, has its own rules, systems and symbols. Language is used to express these mathematical indicators (Galligan, 2021). Since language and thought date back to the creation of humans (Başerer & Duman, 2016), the foundations of mathematics are found in the history of thought and language. However, mathematics also determines the thought (Baki, 2008). As an action turns into a thought in the human mind (Mengüşoğlu, 1988; Taşdelen, 2012), it is manifested in various forms (symbols, codes, visuals, signs, words, numbers, etc.) through language. These indicators acquire a different dimension through meaning in the minds of individuals. Language allows the human to be and aware of one's self (Uygur, 1984). Objects, entities, and phenomena, animate or inanimate things in the external world acquire a value with semantics. According to Kabael and Ata-Baran (2016), the semantic and semiotic structures of a language, namely vocabulary semantics and syntax, and the "universality of language" (Chomsky, 1965, 1972) demonstrate the significance of comprehension in language and mathematics. This reveals the significance of mathematics (O'Halloran, 2015), a versatile semiotic, in the creation, production, and assignment of semantics (Guiraud, 1994). Similar to thought and cognition, semantics is directly associated with language and mathematics. The universality of language and the universality of mathematics (Waller & Flood, 2016), according to Chomsky (1965, 1972), are associated with semantics. Thus, it could be suggested that thought, meaning, language, learning and mathematics are correlated, and these concepts or disciplines complement one another in several aspects. Therefore, the correlation between language and mathematics is prioritized and it is important to scrutinize the correlation between comprehension and learning, and comprehension and learning levels based on various variables. Since language is the foundation of comprehension and learning, it is also the instrument of successful comprehension and learning.

Language not only reflects the meaning but also knowledge, ideas and perspectives (Clarke et al., 2014). Thus, language entails reading and making sense of the visible and invisible world. Reading is the evaluation and interpretation of the signs, symbols, codes, letters and words by the mind (Akyol, 2014; Harris & Sipay, 1990; Özbay, 2006; Ülper, 2010). In other words, reading is an effort to assign a meaning based on the coordinated operation of various affective and cognitive elements such as sight, attention, focus, perception, remembrance, association, analysis and interpretation (Karatay, 2010, p. 459).

Reading entails coding written symbols by establishing an association between sounds and letters and the meaning of the words and sentences that would then be memorized (Ehri, 2005). Since reading is a cognitive process, it is important to scrutinize the analysis and comprehension dimensions of reading (Wagner et al., 2009). Thus, coding, analysis and evaluation emphasize the significance of comprehension in reading. Reading, a basic language skill, is an effective and complex activity that requires comprehension. Therefore, reading and comprehension are two closely associated cognitive processes. According to Flick & Lederman (2002), reading comprehension and mathematics have common features such as "using past knowledge", "making inferences" and "making sense". According to these authors, individuals (students) who are successful in reading comprehension could interpret math problems better. To accurately read a mathematical text or problem, the issue should be perceived with a holistic approach (Noonan, 1990). This perception is directly associated with mathematics as it requires metacognitive skills. Bloom (1995, p. 60) stated that there were significant correlations between reading comprehension, mathematics and other sciences. Thus, language and mathematics are important for the children to acquire reading comprehension, employment of the concepts, problem-reading (expression of the observations by thinking), detailing a problem, or acquisition of the outcome based on the desired solution (Keşan et al., 2008). Reading comprehension is a physical process in the first stage and continues as a mental and complex process. When the reader reads a text for the first time, she or he creates a scheme of thought that leads to a meaning. The reader employs the language, language indicators and various visuals during the analysis of the meaning of the sentences and words. In other words, words, phrases, syntax, and other related elements between the sentences serve as instruments in the reading-comprehension process (Ahmadi, 2017; Mohseni-Takaloo & Ahmadi, 2017). It could be suggested that visualization, formulation and accurate inferences of the text (question/problem) based on analysis are the physical and mental aspects of reading comprehension in mathematics. These physical and mental elements are present in every mathematical text (question/problem). According to Kintsch and Kintsch, (2005, p. 83), reading entails code decoding (interpretation, vocalization), leading to a comprehensible meaning. Thus, decoding and comprehensible meaning are possible by mathematical expression. Reading comprehension is not only the reader's (individual's) interpretation of a written product, but it is also affected by life, academic achievements and several other factors associated with the individual. Reading comprehension is a complex process where various intellectual skills make sense of and organize the text (Oakhill et al., 2019).

Reading comprehension is among the basic language skills that require the comprehension of the details and making sense of a written product/text (material, visual, etc.) (Rose et al., 2000). Reading comprehension is a highly complex and high-level comprehension process that includes several cognitive elements (Grabe & Stoller, 2002). Although reading comprehension includes finding the meaning of a written text, thinking about the text, investigating the reasons, drawing conclusions and analysis, it also includes intellectual activities such as review, selection, decision-making, translation, interpretation, analysis-synthesis and evaluation (Güneş, 2004). Thus, comprehension is a process that occurs before learning and the information should be successfully comprehended before learning. According to Freitag (1997), individuals with high comprehension skills solve mathematical problems better and are successful in mathematics. In reading comprehension or problem-solving, the following the steps determined by Polya (1957) that entail understanding the problem, planning the solution, implementing the plan and evaluation would improve the comprehension of the problem. The relevant literature emphasized that the problem-solving steps indicated by Polya (1957) should be followed in mathematical problem-solving. The analysis of Polya's problem solving steps would reveal that the problem could be solved and learning could be achieved by reading comprehension and then by transforming the meaning into action. Based on the cognitive aspect of learning, the stages of learning (Grabe, 2009) and the reading-comprehension process and its properties (Karatay, 2010; Sever, 1995), it could be suggested that reading-comprehension and learning are parallel processes. Learning includes processes where the data are coded, stored and remembered (recalled) in the mind (Açıkgöz, 2003; Muzzio et al., 2009). Learned knowledge becomes permanent when it is transformed into images, symbols and codes and recorded in the mind (Senemoğlu, 2004; Ün, 1984). The sub-steps of comprehension included in learning should be managed accurately and successfully. Thus, various studies could be conducted to determine the comprehension levels where mislearning originates. According to Gambrell et al., (2002) reading comprehension entails a reader (student or individual) to assign a meaning to a written text. In short, learning is to make sense of a problem. Making sense of learning is the materialization of learning through interpretation, evaluation, analysis, inference and with certain indicators (numbers, shapes, symbols, codes, etc.).

The Gap in the Literature

Based on the structural (Börekçi, 2015, Gemalmaz, 2010) and semantic aspects (Aksan, 2009) of the language, the accurate comprehension of a text or a mathematical problem occurs through language. The mathematical aspect of language and the linguistic aspect of mathematics are effective factors that dominate the correlation between comprehension and learning. The review of literature on language and mathematics revealed that majority of the studies were theoretical (Ní Ríordáin & O'Donoghue, 2009; O'Halloran, 2015; Wilkinson, 2018). In a meta-analysis of the studies on the correlation between language and mathematics, it was observed that the correlation between the two disciplines was discussed theoretically (Peng et al., 2020). Linguistics literature on the correlation between language and mathematics attempted to materialize the correlation between the disciplines based on exemplary language structures and sentences (Chomsky, 1957, 1965; Harris, 1968; Gemalmaz, 2010). In studies on the correlation between language-mathematics (Merz et al., 2015; Pind et al., 2003) and comprehension-learning (Epçaçan, 2009; Janzen, 2003), the predictive relation between the disciplines was not investigated. Furthermore, studies demonstrated that the correlation between lingual comprehension (Gadamer, 2006) and mathematical comprehension (Tatar & Soylu, 2006) plays a key role in the academic achievement of the students. It was reported that students with reading comprehension skills were more successful in mathematics (Tatar & Soylu, 2006).

Objectives and Research Questions

The present study aimed to determine the correlation between language and mathematics comprehension and learning based on language and mathematics questions. Based on this objective, the following research questions were determined:

1. Does language comprehension (Turkish) predict mathematics comprehension, or vice versa?
2. What is the students' level of language and mathematics comprehension?
3. What are the views of the students on language (Turkish) and mathematics questions?

METHOD

This section includes information on the design, study group, data collection instruments, data analysis, validity and reliability of the instruments.

Research Design

This study was conducted with the exploratory sequential design, a mixed research method where both quantitative and qualitative techniques are employed. In the first stage, quantitative data were collected, and the findings were analyzed. In the second stage, qualitative findings were employed to discuss the quantitative data (Creswell, 2009). In explanatory design, qualitative data are employed to analyze quantitative data (Creswell & Plano-Clark, 2015). The details on the quantitative and qualitative data collection and analysis are presented in Figure 1.

The quantitative section of the study is a correlational design in which we utilized linear regression and direction dependence analysis (Wiedermann et al., 2020). The estimation model was employed to determine the magnitude of the effect between language (Turkish) and mathematics, and the likely direction of the prediction (whether language comprehension leads mathematics comprehension or vice versa).

The qualitative study was conducted with the case study model. A case study entails in-depth investigation of a limited system based on a large and comprehensive dataset (Creswell, 2009). The qualitative study data included multiple case study findings. Multiple or collective case research is used to investigate more than one case to study a specific case (Bogdan & Biklen, 2007; Merriam, 2009; Yin, 2018). The first research questions was determined as "Does language (Turkish) predict mathematical achievements, or mathematics predict language achievements?" The second research problem aimed to determine the reading comprehension and learning levels of the students based on language (Turkish) and mathematical comprehension questions. The third research problem aimed to determine whether comprehension was transformed into learning. To research these multiple problems, the views of the students on language (Turkish) and mathematics questions were collected after the test.

In the first stage, for quantitative data analysis, 62 8^{th} grade students took a language (Turkish) and mathematics test that included 14 reading comprehension questions. The students were allowed 25 minutes to complete each test. As for the qualitative data analysis, nine students were interviewed face-to-face after the students completed both the language (Turkish) and mathematics tests. Results from quantitative and qualitative analysis were crosschecked, consistencies and discrepancies were reported.

Study Group

The current study was conducted in a junior high school in a province in the Southeastern Anatolia Region in Turkey. The study sample included 8^{th} grade students. It was assumed that several comprehension topics on language (Turkish) and mathematics curricula in 8^{th} grade was are instructed. At the beginning of the study, power analysis results suggested that 85 participants were required to identify a moderate impact (f^2=.15 or R^2 =.13) with a power of 80% and Type I error of 5% based on two-tailed hypothesis testing. We calculated this figure with the pwrss.f.reg() function in the pwrss R software (Bulus, 2023). However, we were able to collect data from 62 students with which we can detect an effect as small

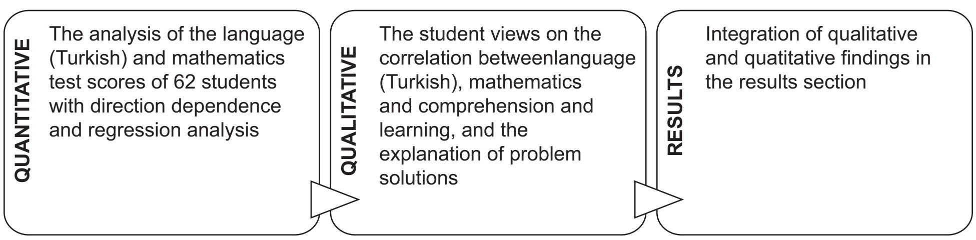

Figure 1. The research process

as f^2 = .212 or R^2 = .175 with a power of 80% and Type I error of 5% based on two-tailed hypothesis testing. Results would be considered ambiguous if estimated effects from the linear regression is smaller than minimum detectable effects (Bulus, 2022; Bulus & Dong, 2021) mentioned earlier and they are also not statistically significant (Bulus & Koyuncu, 2021). Of the 62 8th grade students, 28 of them were male, 35 of them were female. The academic achievement levels of the 62 students were categorized low, medium and high based on independent measurements (pilot achievement tests administered by the school).

Data Collection Instruments

The quantitative study data were collected with the achievement test and the rubric. The qualitative study data were collected with on-structured interview form.

Construction of language and mathematics achievement tests

The quantitative study data were collected with achievement tests. Achievement tests included new generation questions developed by the Turkish Ministry of National Education (2019). The test questions were selected by teachers with a master's degree or PhD or who were master's or PhD candidates in language (Turkish) and mathematics education. To guide selection of new generation comprehension questions, a form that outlines item inclusion criteria was developed for language (Turkish) and mathematics teachers. This form was sent to language (Turkish) and mathematics teachers, and they were asked to select three comprehension questions. Teachers were also asked whether selected questions were comprehension questions and why they selected those questions. Ten language (Turkish) and 10 mathematics teachers selected 60 questions in total.

After the questions were received from the teachers, the authors developed the 14-item achievement test, by selecting 7 questions from the pool of 30 language (Turkish) questions and 7 questions from the pool of 30 mathematics questions. Three language (Turkish) questions were based on visuals such as figures, graphics and tables, and the rest were reading comprehension questions, 4 math questions were based on visuals such as figures, graphics and tables, and the remaining 3 questions were comprehension questions.

The expert views on whether the questions in the language (Turkish) and mathematics test were based on comprehension (strongly agree = 3, agree = 2, disagree = 1, strongly disagree = 0) are presented in Table 1. Figures in the table are based on the average of two expert raters.

The language (Turkish) and mathematics questions in the achievement test were solved by the experts before the application. Turkish language experts solved the language questions and mathematics experts solved the mathematics questions and detailed the solutions in writing. Answers for the language (Turkish) and mathematics questions were scored by the authors and the interrater agreement coefficient was calculated. Thus, high agreement was determined between the raters in the language (Turkish) test (r =.95) and mathematics test (r =.98).

Table 1. The adequacy of the achievement test

	Expert Opinion	
Item	**Language (Turkish)**	**Mathematics**
1	3	3
2	3	3
3	2.50	3
4	3	2.50
5	2.50	2.50
6	2.50	3
7	2.25	3

Analytic rubric

After the language (Turkish) and mathematics tests were finalized, a rubric was developed to evaluate student's answers to test questions. Students were asked to solve the questions and detail their rationale for the answers in written form. The rubric was developed by experts in the language (Turkish) and mathematics education field and aimed to rate the comprehension and learning components in the solutions. The analytic rubric had 4 levels. The lowest possible score in the rubric was 1 meaning low comprehension and learning, whereas the highest score was 4 meaning high comprehension and learning. Students' comprehension level was measured by their correct answers, and their learning level was measured by their ability to explain the problem-solving stages in writing.

Non-structured interviews

The qualitative data were collected with non-structured interview transcriptions. The face-to-face interviews focused on the relations between language-mathematics and comprehension-learning based on the language (Turkish) and mathematics test questions. Face-to-face interviews were conducted with nine students with across all achievement levels (three in each of the low, moderate and high achievement levels). The interviews were recorded, and were later transcribed.

Data Analysis

Quantitative data analysis

After mathematics comprehension/learning (MCL) and Turkish language comprehension/learning (LCL) scales (consisting of 14 items together) were applied to 62 participants, two experts in the fields of language (Turkish) and mathematics rated solutions in terms of comprehension and learning using the analytic rubric mentioned earlier. The average of the two raters were recorded in a dataset along with achievement levels of students determined by an independent achievement test administered before this study. Before embarking on further analysis, measurement models were inspected. Measurement models provide evidence as to whether MCL and LCL scales, each consisting of seven items, form a meaningful construct for comprehension and learning. After establishing the measurement models, the goal was to inspect the relationship between language (Turkish) and mathematics in terms of comprehension and learning. Using factor scores obtained from the measurement models, the magnitude of the effect between MCL and LCL was determined using linear regression and the likely direction of the effect was determined using DDA (Wiedermann et al., 2020). Nonetheless, existing literature suggests relationship between the two domains may differ across different achievement levels. Thus, additional analysis inspected whether the magnitude and the likely direction of the effect changes across achievement levels.

Measurement models

Exploratory factor analysis (EFA) findings suggested that both MCL and LCL scales included two dimensions based on the conceptual scale framework. We used fa() function in psych R software (Revelle, 2018) to conduct EFA. Both scales were developed to include items that represent the textual comprehension (MCL1, MCL2, and MCL3 items in MCL scale; LCL1, LCL2, LCL3, and LCL7 items in LCL scale) and visual comprehension dimensions (MCL4, MCL5, MCL6, and MCL7 in MCL scale; LCL4, LCL5, and LCL6 in LCL scale). EFA results suggested that the items were clearly suitable for their respective dimensions.

In the next step, the measurement models in the EFA step were constructed as confirmatory factor analysis (CFA) models to score the dimensions. We used cfa () function in lavaan R software (Rosseel, 2012) to conduct the CFA. However, there were some convergence problems, indicating that EFA solutions may not be as optimal as planned. Although the MCL scale initially included four Likert-type categories, the model fit for one- and two-factor solutions was unacceptable (see Table 1). We ended up collapsing the categories. Item MCL1 was removed from further analysis since it led to convergence problems in the MCL measurement model due to the high correlation between MCL1 and MCL2. For the remaining items, a model with two categories, two factors, and correlated errors between the items MCL6 and MCL7 produced good data-model fit indices (χ^2/df = 1.08, CFI =.99, TLI =.99, RMSEA =.04, and SRMR =.14). In the MCL scale, Cronbach's α reliability coefficient was.72 for one factor named visual comprehension and.79 for the other factor named textual comprehension.

Similarly, item LCL1 was removed from further analysis since it led to convergence problems in the LCL measurement model due to the high correlation between LCL1 and LCL2. For the LCL scale, a model with four categories, one factor, and correlated errors between the items LCL2 and LCL3 produced good data-model fit indices (/df = 0.33, CFI = 1.00, TLI = 1.00, RMSEA = 0, and SRMR =.05). Cronbach's reliability coefficient was.75 for the LCL scale. Factor scores were calculated with the retained models presented in Table 2. These models with high data-model fit indices were kept (shown in bold in Table 2). Although the model-data fit for the two-factor solution was as good as the fit for the retained model, the correlation between the two factors (visual and textual comprehension) was above.95. The high correlation between these two factors suggested that a one-factor solution was a better option. The fact that there was a high correlation between certain items in MCL and LCL models was due to similar questions (see Residual Constraints row in Table 2).

Direction dependence analysis

Recently, several scholars developed statistical tests to determine directional dependence in linear regression models based on the non-normality of the observed variables and residuals (Wiedermann et al., 2020; Wiedermann & Li, 2018; Wiedermann & von Eye, 2015; Wiedermann & Sebastian, 2020). Statistical tests for direction dependence analysis (DDA) revolve around the observed variables' distributional properties, residuals, and independence of predictors and errors (Wiedermann et al., 2020). The statistical tests are conducted with two competing models: a target and an alternative model, where the predictor and the outcome are switched.

Table 2. Measurement models

	Math Comprehension (MCL) Scale			**Turkish Language Comprehension (LCL.) Scale**			
Measurement Levels	Four	Four	Forced Two	Four	Four	Four	Four
Number of Factors	One	Two	Two	One	One	Two	Two
	45.23	28.6	9.73	17.09	2.62	17.45	2.63
$\chi^2 df$	9	10	9	9	8	9	9
χ^2/df	5.03	2.86	1.08	1.90	0.33	1.94	0.29
CFI	0.86	0.93	0.99	0.95	1	0.95	1
TLI	0.76	0.89	0.99	0.91	1	0.92	1
RMSEA	0.26	0.18	0.04	0.12	0	0.11	0
SRMR	0.15	0.13	0.14	0.11	0.05	0.11	0.05
Cronbach's α	0.75	(0.71, 0.79)	(0.72, 0.79)	0.75	0.75	(0.60, 0.75)	(0.60, 0.75)
Residual Constraints	No	No	MCL6 ~~ MCL7	No	LCL2 ~~ LCL3	No	LCL2 ~~ LCL3
Retain the model?	No	No	Yes	No	Yes	No	No

DDA was conducted on the factor scores obtained from the CFA procedures described in the Measurement Models section. Furthermore, since the MCL model had two factors, these scores were summed to get an overall score. Additional DDA models were run with two sets of factor scores for the MCL sub-scales (visual and textual comprehension); however, the results did not change substantially. To inspect the likely direction of the effect between MCL and LCL scales, the DDA R software (Wiedermann & Li, 2019) available at http://www.ddaproject.com was used. The lm() function in R (R Core Team, 2021) was used to obtain regression coefficients and R-squares to determine the magnitude of the effect between MCL and LCL scales.

Qualitative data analysis

The interview transcripts were analyzed. The data were analyzed with content analysis and classified into main and sub-themes. In addition to this classification, the views of certain participants were directly quoted in the paper. The interviews were conducted with nine participants, three students in each achievement level (low, moderate, and high). Students with low achievement were coded with "LL1", "LL2", and "LL3", students with moderate achievement were coded with "ML1", "ML2", and "ML3", and students with high achievement were coded with "HL1", HL2, and "HL3".

The role of the authors

The first author has graduate and Ph.D. degrees in education, Turkish instruction as a native language, and Turkish instruction as a foreign language. The first author published several articles and book chapters on the field. The second author has undergraduate, graduate, and Ph.D. degrees in mathematics instruction and conducted several studies on mathematics education and teaching. The third author has master's and doctorate degrees in measurement and evaluation and published several papers. Before the study, all three authors reported that comprehension was essential in language and mathematics. Then, three authors reviewed the previous studies on the correlation between language-mathematics and reading-learning (Ní Ríordáin & O'Donoghue, 2009; O'Halloran, 2015; Nesher & Katriel, 1986; Wilkinson, 2018) and established the framework of the present study.

RESULTS

To address the first research problem, "Does language (Turkish) predict mathematical achievements or mathematics predict language achievements?" linear regression and DDA results are presented in what follows. The linear regression results to determine the magnitude of the effect between LCL and MCL and the DDA analysis results to determine the likely direction of the effect are presented in Table 3.

Results suggest that the alternative model was more likely to hold for the overall sample (MCL <- LCL). Hilbert-Schmidt independence criterion and non-linear correlation tests (not shown) suggested that it was more likely for the MCL to be the outcome and for L.C.L. to be the predictor. The LCL had a moderate catalytic effect on the MCL (Standardized $\beta = 0.48$, p <. 001). In other words, high LCL scores are more likely to lead to high MCL scores. In contrast, for the overall model, based on the skewness of the residuals in target and alternative models, LCL was more likely to be the outcome. The difference between skewness figures was statistically significant (0.72 at 95% CI [0.11, 1.38]). Thus, further sub-group analysis could resolve this controversy based on achievement levels.

DDA tests were inconclusive for the low-achievement subgroup. The directionality of the effect for low-achievement students could not be determined. Interesting evidence emerged for the moderate achievement subgroup. Contrary to the whole sample, both Hilbert-Schmidt independence criterion and homoscedasticity tests favored the target model (LCL <- MCL), suggesting that it was more likely for the LCL to be the outcome and for the MCL to be the predictor. The MCL had a small to moderate inhibitory effect on the LCL (Standardized $\beta = -0.30$, n.s.) in the moderate achievement subgroup. As for the high achievement subgroup, homoscedasticity tests favored the alternative model (MCL <- L.C.L.), suggesting that it was more likely for the MCL to be the outcome and for the L.C.L. to be the predictor. Other tests were inconclusive. LCL had a small to moderate catalytic effect on MCL (Standardized $\beta = 0.26$, n.s.). None of the effects were statistically significant in the subgroup analysis; however, this was likely due to the small sample size. It should be noted that the power analysis targeted the whole model. Thus, standardized effect sizes were more informative.

To address the second research problem, "What is the students' level of comprehension in language and mathematics questions?", average expert ratings were cross-tabulated by items and achievement levels. Numbers in the cells are averages of students in each achievement level category.

As seen in Table 4, average ratings of comprehension-learning for students who answered the LCL and MCL achievement tests incorrectly were lower when compared to those who answered the questions correctly. Thus, as the achievement level increased, comprehension-learning also increased, demonstrating a direct relationship between achievement and comprehension-learning. Average ratings of students on LCL and MCL achievement tests varied between 1.00 and 2.00. This corresponds to the comprehension-learning level labeled "making sense/finding the meaning."

To address the third research problem, "What are the students' views on language (Turkish) and mathematics questions?" results of the structured interviews are presented below.

As seen in Table 5, the main common theme in the language (Turkish) and mathematics questions was "reading comprehension." When the sub-theme in the language (Turkish) comprehension was poor, textual comprehension, association, evaluation, and comprehension were moderate, and logical reasoning and abstract thinking was high. When the sub-themes in mathematics comprehension were poor,

Table 3. Direction dependence analysis

Sample	Direction Dependence Test	Target Model LCL <- MCL	Alternative Model MCL <- LCL	Interpretation
Overall (*N*=62)	Residual distribution			Evidence favors the alternative model.
	Skewness	0.05	-0.77*	
	Kurtosis	-0.26	0.91	
	Hilbert-Schmidt independence criterion			
	Gamma approximation	0.92***	0.63*	
	Bootstrap approximation	0.92**	0.47	
	Homoscedasticity tests			
	Breusch-Pagan test	0.65	0.50	
	Robust Breusch-Pagan test	0.75	0.34	
	Standardized Regression Coefficient	0.48***		
	Adjusted R-squared	0.22		
Low-achieving (*N*=21)	Residual distribution			Inconclusive results. Trivial effect.
	Skewness	-0.63	-0.64	
	Kurtosis	-1.04	-0.52	
	Hilbert-Schmidt independence criterion			
	Gamma approximation	0.69*	0.65*	
	Bootstrap approximation	0.69*	0.54*	
	Homoscedasticity tests			
	Breusch-Pagan test	0.15	2.37	
	Robust Breusch-Pagan test	0.32	3.71	
	Standardized Regression Coefficient	0.08		
	Adjusted R-squared	0.01		
Medium-achieving (*N*=21)	Residual distribution			Evidence favors the target model.
	Skewness	-0.36	-0.59	
	Kurtosis	-0.33	-0.42	
	Hilbert-Schmidt independence criterion			
	Gamma approximation	0.2	0.72**	
	Bootstrap approximation	0.2	0.55	
	Homoscedasticity tests			
	Breusch-Pagan test	0.82	3.90*	
	Robust Breusch-Pagan test	0.98	4.95*	
	Standardized Regression Coefficient	-0.30		
	Adjusted R-squared	0.09		
High-achieving (*N*=20)	Residual distribution			Evidence favors the alternative model.
	Skewness	-0.18	0.54	
	Kurtosis	-0.04	-1.21	
	Hilbert-Schmidt independence criterion			
	Gamma approximation	0.67*	1.03***	
	Bootstrap approximation	0.67*	0.98**	
	Homoscedasticity tests			
	Breusch-Pagan test	5.61*	0.01	
	Robust Breusch -Pagan test	5.72*	0.02	
	Standardized Regression Coefficient	0.26		
	Adjusted R-squared	0.07		

*p<.05; **p<.01; ***p<.001. MCL: Math Comprehension. LCL: Turkish Language Comprehension.

Table 4. Average comprehension scores of the students based on achievement level and response category

Scale	Achievement Level	Response Category	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7
Turkish Language Comprehension (LCL.)	Low	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		1	1.31	1.22	1.76	1.00	1.00	1.00	1.44
	Moderate	0	*NA*	1.00	*NA*	*NA*	1.06	1.00	1.00
		1	1.14	1.12	1.62	1.19	1.00	1.50	1.45
	High	0	*NA*	1.20	*NA*	*NA*	1.30	1.00	2.00
		1	1.40	1.53	1.65	1.35	1.60	1.71	1.35
Math Comprehension (MCL)	Low	0	1.00	1.00	1.67	1.33	1.12	1.00	1.17
		1	1.28	1.31	2.00	1.00	1.20	1.00	1.00
	Moderate	0	1.00	1.00	2.00	1.73	1.33	1.29	1.00
		1	1.37	1.28	1.59	1.70	1.47	1.29	1.14
	High	0	2.00	2.00	1.33	1.83	1.00	1.67	*NA*
		1	1.74	1.71	1.88	1.79	1.53	2.00	1.70

NA: Not available.

Table 5. Themes and sub-themes about language (Turkish) and mathematics comprehension questions

Scale	Achievement Level	Theme	Sub-theme
Turkish Language Comprehension (LCL.)	Low	Reading-comprehension	Textual comprehension
	Moderate	Reading-comprehension	Textual comprehension
			Attribution
			Evaluation
			Interpretation
	High	Reading-comprehension	Logical thinking
			Abstract thinking
Math Comprehension (MCL)	Low	Reading-comprehension	Problem identification
	Moderate	Reading-comprehension	Understanding the problem
			Attribution
			Evaluation
	High	Reading-comprehension	Logical reasoning
			Abstract thinking
			Synthesis

understanding the problem, evaluation, and association were moderate, whereas logical reasoning, abstract thinking, and synthesis were high. The views of certain participants on the questions based on language (Turkish) and mathematics comprehension are as follows:

> *"I thought I understood the Turkish questions, but I misunderstood them"* (LL1).
>
> *The math questions were mostly on logical comprehension and interpretation.* (LL1).
>
> *Solving Turkish questions correctly is also about comprehension. It is necessary to make sense and interpret. So, we need to interpret the knowledge* (LL2).
>
> *I read the math questions, but I could not understand. I could not associate the shape questions. I could not explain them because I did not understand the questions. My inability to understand the math questions stemmed from primary school* (LL2).
>
> *When I was solving Turkish questions, first, I tried to understand them. Then I solved them by considering the sentence's relationship and explained the questions that way. I also answered the visual questions correctly by evaluation. I made an inference* (ML1).
>
> *I understood the math questions, but the questions were hard to understand. This suggests that it is difficult actually to turn understanding into learning. This depends on whether the person has solved the questions before. I employed attribution and estimation in solving the problems* (ML1).
>
> *After reading and understanding the questions in Turkish, I answered them. Because there was a correlation between sentences, this relationship connection led me to the right answer. I tried to understand the meaning of some questions or answered them based on the choices.* (ML2).
>
> *Explaining math questions, I mean, detailing them, was a little longer. It seemed long to understand and explain how I got the right answer. I did not solve the math problems because I generally did not understand them (ML2)*

I solved Turkish questions based on the meaning. They were based on interpretation (HL1).
Math questions were based on thinking and interpretation (HL1).
It is necessary to find the main idea in solving mathematics and Turkish questions (HL2).
Math questions require some thinking, and I found them associated with real life. So, I made an inference. I tried to understand the logic of the problems. And then I solved them based on the choices. I could not determine what to do on the questions I did not infer (HL2).

DISCUSSION AND CONCLUSION

Language is an advanced system constructed with sounds that mediate the expression of emotions and ideas with certain indicators such as letters, shapes, strings, symbols, and codes based on certain rules to allow communication among individuals (Aksan, 2009; Barre et al., 2011; Catts et al., 2006; Gencan, 2007; Hengirmen, 2007; Vardar, 2007), while mathematics is a universal language (Adoniou & Qing 2014; Waller & Flood, 2016) that provides communication between individuals using terms, concepts, symbols, and grammar (Cirillo et al., 2010). The present study findings demonstrated that language comprehension-learning was effective in mathematical comprehension-learning (Standardized $\beta = 0.48$). Thus, academic achievement in mathematics depends on language skills.

Like mathematical symbols and drawings, language plays a crucial role in formulating and expressing mathematical ideas and operations as a bridge between abstract and concrete representations. Language encompasses all aspects of human life and reflects concrete ideas (Wittgenstein, 2008), and mathematics, which represents ideas in symbols, are two interrelated disciplines (O'Halloran, 2015). Mathematics is a language with unique symbols and terminology, where there are meaningful relationships between these (Stone, 2013).

The present study findings that the direction of the causality between the achievements in language and mathematics varied and were associated with the multi-semiotic character of language and mathematics. For example, students with a rich vocabulary exhibit high mathematical achievements (Koç et al., 2002), while students with a low language (Turkish) achievement exhibit low mathematical achievements (Albayrak & Erkal, 2003), and the correlation between language (Turkish) and mathematics affects academic achievement. Accurate use of language also affects mathematical learning (Taşkın, 2013). This is because mathematics is not language-independent and is a multi-semiotic science (O'Halloran, 2015; Ní Ríordáin & O'Donoghue, 2009).

Research on the correlation between language and mathematics reported different findings. Certain studies concluded that there was a strong correlation between language and mathematics ($r \geq .70$; Merz et al., 2015; Pind et al., 2003), while others reported that the exact correlation was weak ($r < .20$; Mellard et al., 2015; Mestre, 1981). The present study findings demonstrated that language and mathematics comprehension-learning generally share 48% of the variation. This correlation varied based on low, moderate, and high academic achievement levels. In the study, the shared variance between language and mathematics comprehension-learning was 08% for the low academic achievement group students. This finding was consistent with previous results ($r < .20$; Mellard et al., 2015; Mestre (1981). However, in the study, it was unclear whether language predicted mathematics or mathematics predicted language achievement for those in the low academic achievement group. For moderate educational achievement group students, the shared variance between language and mathematics comprehension-learning was 9%. This was also consistent with the findings Mellard et al. (2015) and Mestre (1981) reported. On the contrary, the shared variance between language and mathematics comprehension-learning was 26% for the high-achievement group.

The present study findings demonstrated that achievement in language generally predicted achievement in mathematics. However, the causality between academic achievements in language and mathematics varied based on the academic achievement levels of the students. In a study on language and mathematics, Peng et al. (2020) reported that the correlation between these disciplines varied based on various variables. The finding that the direction of the causality between language and mathematics differed based on the academic achievement level was consistent with the results reported by Peng et al. (2020). Also, Peng et al. (2020) determined in the meta-analysis that the correlation between these two disciplines was $r = .42$ at 95% CI (.40.,44). In this study, overall, the shared variance between language and mathematics comprehension-learning was 48%. This finding was consistent with the result Peng et al. reported (2020). The present study differed from previous study findings since the likely direction of the effect was scrutinized overall and across achievement levels.

Studies on the correlation between language and mathematics (Merz et al., 2015; Pind et al., 2003; Peng et al., 2020) and students' views in the present study demonstrated the correlation between language and mathematics. For example, the opinions of the participants such as "*Math questions include more logical comprehension and interpretation*" (LL1), "*Math questions are based on thinking and interpretation ability" (HL1), "Responding Turkish questions accurately requires comprehension. It requires substantiation and interpretation. In other words, we need to interpret the information"* (LL2) reflected the intersection between language and mathematics. Thus, since language plays a crucial role in mathematics instruction, language, and mathematics instruction should be planned accordingly.

The quantitative and qualitative study findings revealed that language and mathematics complemented each other, and there was a direct correlation between the two disciplines regarding comprehension and learning. To comprehend a mathematical text or question, it is necessary to understand the whole question (Noonan, 1990). The stages of comprehending a problem include understanding it, making a solution plan, implementing it, and evaluating it (Polya, 1957). The qualitative study data demonstrated that the most prevalent three themes determined based on the views of the students with low, moderate, and high academic achievements were understanding the problem, analysis, and evaluation. The correlation determined between language comprehension/learning and math comprehension/learning in the present study was consistent with the reports by Bloom (1995, s. 60). Furthermore, to ensure the comprehension and employment of the concepts, comprehension and expression of the problem. The desired continuity of the solution (Keşan et al., 2008, pp. 2-3), the language in which the concept or the problem was expressed should be comprehended accurately.

The prevalent sub-themes revealed by the qualitative study data on the language (Turkish) and mathematics questions included comprehension, interpretation and analysis of the text/problem. Reading entails making sense of the codes and their meaning (Kintsch & Kintsch, 2005). Students with high comprehension skills solved mathematical problems better and were more successful (Freitag, 1997, pp. 16-17). Also, the study findings demonstrated that the students' mean language (Turkish) and mathematics achievement score was 2/4. This mean score demonstrated that students were below the moderate level in comprehension and learning.

Based on the qualitative study data, the common central theme in the language (Turkish) and mathematics questions was that these were both based on "reading comprehension." This finding was similar to previous reports. Mathematics is a "multi-semiotic science" (O'Halloran, 2015), and reading, a complex activity requiring comprehension, were standard features of both mathematics and comprehension. According to Flick & Lederman (2002), reading comprehension and mathematics have standard features such as "the employment of prior knowledge, "making inferences," and "making sense." The sub-themes determined in the present study were evaluation, association of knowledge, analysis, interpretation, logical reasoning, abstract thinking, and synthesis. Furthermore, according to students with different academic achievement levels, the solution to a problem in both language (Turkish) and mathematics is based on reading comprehension. Thus, students who are successful in reading comprehension would be successful in both language (Turkish) and mathematics.

Reading comprehension entails making sense of a text. In the current study, students were asked to respond to language (Turkish) and mathematics questions and explain their solutions. The study findings revealed that the learning (explanation) level of the students with low comprehension skills was low, while those with high comprehension levels exhibited higher learning (explanation) levels. The study findings demonstrated a correlation between comprehension and learning. This finding was consistent with the findings Gambrell et al., (2002) reported.

The permanence of the learned knowledge in mind through transformation into images, symbols, and codes (Senemoğlu, 2004; Ün, 1984) reflects the correlation between mathematics and reading comprehension (Flick & Lederman, 2002; Noonan, 1990). It is, furthermore, based on the common properties of language and mathematics that both include codes and symbols (Harley, 1995; Cangelosi, 2011; Miller, 1963; Saussure, 1916; Sapir, 1921; Francis, 1958; Finochiaro, 1974; Cirillo et al., 2010). It could be suggested that language and mathematics determine the correlation between comprehension and learning. The quantitative and qualitative study findings also evidenced a correlation between language comprehension/learning and math comprehension/learning.

Initially, the study aimed to resolve the "Does language (Turkish) predict mathematical achievements or mathematics predict language achievements?" research problem. Then, the correlation between the two disciplines was determined with direction dependence and regression analyses. After the quantitative findings were obtained from these analyses, common themes and sub-themes were determined based on the language (Turkish) and mathematics comprehension questions. These themes were classified based on the academic achievement levels in the language (Turkish) and mathematics (poor, moderate, high).

Based on the DDA conducted on the general student sample, it was revealed that achievement in the language (Turkish) predicted mathematical achievement. However, the predictive power of the language (Turkish) and mathematics varied based on the academic achievement levels of the students (low, moderate, and high).

Based on the DDA findings, it was unclear whether language (Turkish) predicted mathematics or mathematics predicted language (Turkish) in students with low academic achievement. However, DDA findings demonstrated that students with moderate academic success had high mathematics and low language (Turkish) achievement. Also, DDA revealed that the language (Turkish) achievements of the students with high academic achievement predicted their mathematical achievements.

The present study's findings that aimed to determine the correlation between language comprehension/learning and math comprehension/learning revealed that learning improved with comprehension. Based on the language (Turkish) and mathematics comprehension test results, learning increased with comprehension in both disciplines. These findings were similar in students with poor, moderate, and high achievement levels. However, based on the mean comprehension score of the students based on their achievement levels and response categories, the mean comprehension and learning score was 2/4. Thus, it could be suggested that the comprehension and learning achievements of the participants were generally at a moderate level.

The qualitative findings in the present study that aimed to determine the prediction direction of the correlation between language (Turkish) and mathematics were consistent with the quantitative results. The qualitative study findings revealed that the common theme based on the views of the students with poor, moderate, and high achievements in both disciplines was "reading comprehension." This was similar to the finding in the language (Turkish) and mathematics for "reading comprehension." The prevalent sub-theme based on the views of the students with poor academic achievement in the language (Turkish) was textual comprehension. In contrast, textual comprehension, attribution, evaluation, and interpretation were prevalent among the students with moderate achievement, and logical reasoning and abstract thinking were commonplace among the students with high achievement. The prevalent sub-theme based on the views of students with poor academic achievement in mathematics was textual comprehension, which was understanding the problem, attribution, and evaluation among students with moderate achievement, and logical reasoning, abstract thinking, and synthesis among students with high achievement.

The qualitative study findings revealed that language (Turkish) and mathematics-based comprehension questions were categorized based on similar sub-themes in general. According to the students with different academic achievement levels, both language (Turkish) and mathematics had common properties. The qualitative study findings demonstrated that comprehension and learning included the processes of association, evaluation, interpretation, logical reasoning, abstract thinking, and synthesis in both disciplines.

RECOMMENDATIONS

Further extensive studies could be conducted with a longitudinal approach or direction dependence analysis and larger samples to determine the direction of the causality between language and mathematics. Future empirical research could investigate the correlation between comprehension and learning in language and mathematics based on achievement levels (poor, moderate, and high). The difference between comprehension and learning could be determined in different grade levels based on the concrete and abstract developmental periods in language and mathematics. The correlation between language and mathematics could be investigated based on basic language skills and other metacognitive skills associated with basic language skills. The factors that affect the correlation between comprehension and learning could be investigated in future studies.

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